ventricle, and the extravaſation covering the fiſſure in the aorta, exactly marked, as they appeared to,

My Lord,
Your Lordſhip's
moſt obedient
and moſt humble ſervant,

Frank Nicholls.

LII. *Of the Irregularities in the planetary Motions, cauſed by the mutual Attraction of the Planets: In a Letter to* Charles Morton, *M. D. Secretary to the Royal Society, by* Charles Walmeſley, *F. R. S. and Member of the Royal Academy of Sciences at* Berlin, *and of the Inſtitute at* Bologna.

SIR,

Read Dec. 10, 1761.

FInding that the influence, which the primary planets have upon one another, to diſturb mutually their motions, had been but little conſidered, I thought it a ſubject worthy of examination. The force of the ſun, to diſturb the moon's motion, flows from the general principle of *gravitation*, and has been fully aſcertained, both by theory and obſervation; and it follows, from the

ſame

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ſame principle, that all the planets muſt act upon one another, proportionally to the quantities of matter contained in their bulk, and inverſe ratio of the ſquares of their mutual diſtances; but as the quantity of matter contained in each of them, is but ſmall when compared to that of the ſun, ſo their action upon one another, is not ſo ſenſible as that of the ſun upon the moon. Aſtronomers generally contented themſelves with ſolely conſidering thoſe inequalities of the planetary motions, that ariſe from the elliptical figure of their orbits; but as they have been enabled, of late years, by the perfection of their inſtruments, to make obſervations with much more accuracy than before, they have diſcovered other variations, which they have not, indeed, been able yet to ſettle, but which ſeem to be owing to no other cauſe, but the mutual attraction of thoſe celeſtial bodies. In order, therefore, to aſſiſt the aſtronomers in diſtinguiſhing and fixing theſe variations, I ſhall endeavour to calculate their quantity, from the general law of gravitation, and reduce the reſult into tables, that may be conſulted, whenever obſervations are made.

I offer to you, at preſent, the firſt part of ſuch a theory, in which I have chiefly conſidered the effects produced by the actions of the earth and Venus upon each other. But the ſame propoſitions will likewiſe give, by proper ſubſtitutions, the effects of the other planets upon theſe two, or of theſe two upon the others. To obviate, in part, the difficulty of ſuch intricate calculations, I have ſuppoſed the orbits of the earth and Venus to be originally circular, and to ſuffer no other alteration, but what is occaſioned by their mutual attraction, and the attraction of the other planets. Where the forces of two planets are conſiderable, with reſpect to each other, as in the caſe of Jupiter and Saturn, it may be neceſſary, in ſuch computations, to have regard to the excentricity of their orbits; and this may be reſerved for a ſubject of future ſcrutiny. But the ſuppoſing the orbits of the earth and Venus to be circular, may, in the preſent caſe, be admitted, without difficulty, as the forces of theſe two planets are ſo ſmall, and the excentricity of their orbits not conſiderable. On theſe grounds, therefore, I have computed the variations, which are the effects of the earth's action: firſt, the variation of Venus's diſtance from the ſun; ſecondly, that of its place in the ecliptic; thirdly, the retrograde motion of Venus's nodes; and, fourthly, the variation of inclination of its orbit to the plane of the ecliptic.

The ſimilar irregularities in the motion of the earth, occaſioned by its gravitation to Venus, are here likewiſe computed: but it is to be obſerved, that the abſolute quantity of theſe irregularities is not here given, it being impoſſible, at preſent, to do it; becauſe the abſolute force of Venus is not known to us. I have, therefore, ſtated that planet's force by ſuppoſition, and have, accordingly, computed the effects it muſt produce; with the view, that the aſtronomers may compare their obſervations with the motions ſo calculated, and, from thence, diſcover how much the real force differs from that which has been ſuppoſed. But the exact determination of the force of Venus muſt be obtained, by obſervations made on the ſun's place, at ſuch times, when the effect of the other planets is either null or known.

The influence of Venus upon the earth being thus computed, that of the other planets upon the ſame, may likewiſe, hereafter, be conſidered: by which means, the different equations, that are to enter into the ſettling of the ſun's apparent place, will be determined; the change of the poſition of the plane of the earth's orbit will alſo be known; and, conſequently, the alteration that thence ariſes in the obliquity of the ecliptic, and in the longitude and latitude of the fixed ſtars. Theſe matters of ſpeculation are reſerved for another occaſion, in caſe what is here offered ſhould deſerve approbation.

I am glad to have it in my power to preſent you with this teſtimony of my gratitude for paſt favours, and of my reſpect for your diſtinguiſhed merit; and it is with ſincerity, I ſubſcribe myſelf,

SIR,

Your very humble ſervant,

Bath,
Nov. 21, 1761.

Cha. Walmeſley.

De Inæqualitatibus quas in motibus Planetarum generant ipſorum in ſe invicem actiones.

QUoniam in theoriæ hujus decurſu frequens erit uſus fluentium quæ arcubus circuli, vel eorum ſinibus, coſinibus, et ſinibus verſis, exprimuntur, idcircò lemma ſequens, quod alibi olim tradidi, lubet hîc apponere.

LEMMA.

LEMMA.

Dato coſinu arcûs cujuſvis, invenire coſinum et ſinum arcûs alterius qui ſit ad priorem in ratione λ ad 1.

Detur c coſinus arcûs A ad radium 1, et ſit arcus $B = \lambda A$, cujus coſinus dicatur t; eritque, ut notum eſt, $\dot{A} = \frac{-\dot{c}}{\sqrt{1-cc}}$, atque $\dot{B} = \lambda \dot{A} = \frac{-\dot{t}}{\sqrt{1-tt}}$. Ponatur $c = \frac{1+xx}{2x}$, et $t = \frac{1+yy}{2y}$, fietque $\dot{A} = \frac{\dot{x}}{x\sqrt{-1}}$, $\dot{B} = \frac{\dot{y}}{y\sqrt{-1}}$: ſed eſt $\dot{A} . \dot{B} :: 1 . \lambda$, adeoque $\frac{\lambda \dot{x}}{x} = \frac{\dot{y}}{y}$; unde log. $x^{\lambda} =$ log. y, et $x^{\lambda} = y$. Verùm æquationes $c = \frac{1+xx}{2x}$ et $t = \frac{1+yy}{2y}$ dant $x = c + \sqrt{cc-1}$, $x = c - \sqrt{cc-1}$, et $y = t + \sqrt{tt-1}$, $y = t - \sqrt{tt-1}$; unde eſt $x^{\lambda} = t \pm \sqrt{tt-1} = \overline{c \pm \sqrt{cc-1}}^{\lambda}$, atque inde $2t = \overline{c + \sqrt{cc-1}}^{\lambda} + \overline{c - \sqrt{cc-1}}^{\lambda}$. Fiat igitur $c + \sqrt{cc-1} = l$, et $c - \sqrt{cc-1} = m$, eritque $lm = 1$, et $c =$ coſ. $A = \frac{l+m}{2}$, et ſin. $A = \frac{l-m}{2}\sqrt{-1}$; atque inde $t =$ coſ. $B = \frac{l^{\lambda} + m^{\lambda}}{2}$, et ſin. $B = \frac{l^{\lambda} - m^{\lambda}}{2}\sqrt{-1}$.

Itaque in circulo, cujus radius eſt 1, ſi duorum arcuum vel angulorum A et B alteruter B ſit ad alterum A ut numerus quilibet λ ad 1, et ponatur coſ. $A = \frac{l+m}{2}$, exiſtente $lm = 1$, erit ſin. A

=

$= \frac{l-m}{2}\sqrt{-1}$, atque coſ. B $=$ coſs $\lambda A = \frac{l^{\lambda}+m^{\lambda}}{2}$, et ſin. B $=$ ſin. $\lambda A = \frac{l^{\lambda}-m^{\lambda}}{2}\sqrt{-1}$. Q. E. I.

COROLL. I.

Hinc habetur coſ. A $\times$ coſ. B $= \frac{l+m}{2}\times\frac{l^{\lambda}+m^{\lambda}}{2} = \frac{l^{\lambda+1}+m^{\lambda+1}}{4}+\frac{l^{\lambda-1}+m^{\lambda-1}}{4}$; ſed, quemadmodùm per hoc lemma eſt $\frac{l^{\lambda}+m^{\lambda}}{2} =$ coſ. λA, erit $\frac{l^{\lambda+1}+m^{\lambda+1}}{2} =$ coſ. $\overline{\lambda+1}\times A =$ coſ. $A+B$, atque $\frac{l^{\lambda-1}+m^{\lambda-1}}{2} =$ coſ. $\overline{\lambda-1}\times A =$ coſ. $\overline{B-A}$, adeoque coſ. A $\times$ coſ. B $= \frac{1}{2}$ coſ. $\overline{A+B} + \frac{1}{2}$ coſ. $\overline{B-A}$.

Atque hoc calculi methodo facilè eruuntur ſequentes formulæ pro duobus angulis A et B, advertendo eſſe coſ. $\overline{B-A} =$ coſ. $\overline{A-B}$, ſin. $\overline{B-A} = -$ ſin. $\overline{A-B}$, et coſ. $0 = 1$.

1°. Coſ. A $\times$ coſ. B $= \frac{1}{2}$ coſ. $\overline{A+B} + \frac{1}{2}$ coſ. $\overline{A-B}$.
2°. Sin. A $\times$ ſin. B $= -\frac{1}{2}$ coſ. $\overline{A+B} + \frac{1}{2}$ coſ. $\overline{A-B}$.
3°. Sin. A $\times$ coſ. B $= \frac{1}{2}$ ſin. $\overline{A+B} + \frac{1}{2}$ ſin. $\overline{A-B}$.

Atque ex illis hæ ſequentes eliciuntur,

4°. Coſ. $\overline{A+B} =$ coſ. A $\times$ coſ. B $-$ ſin. A $\times$ ſin. B.
5°. Coſ. $A-B =$ ſin. A $\times$ ſin. B $+$ coſ. A $\times$ coſ. B.
6°. Sin. $A+B =$ ſin. A $\times$ coſ. B $+$ coſ. A $\times$ ſin. B.
7°. Sin. $A-B =$ ſin. A $\times$ coſ. B $-$ coſ. A $\times$ ſin. B.

Tùm ex his valores tangentium haud ægrè derivantur, Quippe cum ſit generatim pro quovis angulo A, $\text{tang. } A = \frac{\text{ſin. } A}{\text{coſ. } A}$, erit $\text{tang. } \overline{A+B} = \frac{\text{ſin. } \overline{A+B}}{\text{coſ. } \overline{A+B}} = \frac{\text{ſin. } A \times \text{coſ. } B + \text{coſ. } A \times \text{ſin. } B}{\text{coſ. } A \times \text{coſ. } B - \text{ſin.} A \times \text{ſin. } B} = \frac{\text{ſin. } A \times \text{coſ. } B + \text{coſ. } A \times \text{ſin. } B}{\text{coſ. } A \times \text{ſin. } B.}$ $\times \frac{\text{coſ. } A \times \text{ſin. } B}{\text{coſ. } A \times \text{coſ. } B - \text{ſin. } A \times \text{ſin. } B} = \overline{\frac{\text{tang. } A}{\text{tang. } B} + 1}$ $\times \frac{1}{\frac{1}{\text{tang. } B} - \text{tang. } A} = \frac{\text{tang. } A + \text{tang. } B}{1 - \text{tang. } A \times \text{tang. } B}$. Simili calculo prodit $\text{tang. } A - B = \frac{\text{tang. } A - \text{tang. } B}{1 + \text{tang. } A \times \text{tang. } B}$. Unde ſtatui poſſunt,

1°. $\text{Tang. } \overline{A+B} = \frac{\text{tang. } A + \text{tang. } B}{1 - \text{tang. } A \times \text{tang. } B}$.

2°. $\text{Tang. } \overline{A-B} = \frac{\text{tang. } A - \text{tang. } B}{1 + \text{tang. } A \times \text{tang. } B}$.

3°. $\text{Tang. } A \times \text{tang. } B = \frac{\text{tang. } \overline{A+B} - \text{tang. } A - \text{tang. } B}{\text{tang. } A+B}$,

vel $\text{tang. } A \times \text{tang. } B = \frac{\text{tang. } A - \text{tang. } B - \text{tang. } \overline{A-B}}{\text{tang. } \overline{A-B}}$.

COROLL. II.

Erat in lemmate $\dot{A} = \frac{\dot{x}}{x\sqrt{-1}}$, unde eſt $A\sqrt{-1} = \log. x$.

Denotet igitur E numerum cujus logarithmus hyperbolicus eſt 1, eritque $E^{A\sqrt{-1}} = x$, et cum ſit $x = c \pm \sqrt{cc-1}$, inde obtinetur $c = \text{coſ. } A = \frac{E^{A\sqrt{-1}} + E^{-A\sqrt{-1}}}{2}$, atque $\text{ſin. } A = \frac{E^{A\sqrt{-1}} - E^{-A\sqrt{-1}}}{2\sqrt{-1}}$.

Sunt qui his ſinuum et coſinuum valoribus potiùs utuntur; verùm ii valores, quos exhibet corollarium præcedens, ſimpliciores ſunt et calculo plerumque aptiores.

COROLL. III.

Quoniam eſt $2 \times \text{coſ. A} = l + m$, erit

$$2^{\lambda} \times \overline{\text{coſ. A}}|^{\lambda} = \begin{cases} l^{\lambda} + \lambda l^{\lambda-1} m + \lambda \times \frac{\lambda - 1}{2} l^{\lambda-2} m^{2} + \lambda \\ \quad \times \frac{\lambda - 1}{2} \times \frac{\lambda - 2}{3} l^{\lambda-3} m^{3} +, \&c. \\ m^{\lambda} + \lambda m^{\lambda-1} l + \lambda \times \frac{\lambda - 1}{2} m^{\lambda-2} l^{2} + \lambda \\ \quad \times \frac{\lambda - 1}{2} \times \frac{\lambda - 2}{3} m^{\lambda-3} l^{3} +, \&c. \end{cases}$$

aſſumendo ſcilicet primos et ultimos terminos homologos ſeriei exprimentis quantitatem $\overline{l + m}|^{\lambda}$: unde, propter $lm = 1$, provenit

$$2^{\lambda-1} \times \overline{\text{coſ. A}}|^{\lambda} = \frac{l^{\lambda} + m^{\lambda}}{2} + \lambda \times \frac{l^{\lambda-2} + m^{\lambda-2}}{2} + \lambda \times \frac{\lambda - 1}{2} \times \frac{l^{\lambda-4} + m^{\lambda-4}}{2} + \lambda \times \frac{\lambda - 1}{2} \times \frac{\lambda - 2}{3} \times \frac{l^{\lambda-6} + m^{\lambda-6}}{2} +, \&c.$$

atque adeò per lemma

$$\overline{\text{coſ. A}}|^{\lambda} = \frac{1}{2^{\lambda-1}} \text{ in coſ. } \lambda\text{A} + \lambda \text{ coſ. } \overline{\lambda - 2} \times \text{A} + \lambda \times \frac{\lambda - 1}{2} \text{coſ. } \overline{\lambda - 4} \times \text{A} + \lambda \times \frac{\lambda - 1}{2} \times \frac{\lambda - 2}{3} \text{ coſ. } \overline{\lambda - 6} \times \text{A} +, \&c.$$

Ubi λ eſt numerus impar, terminus ultimus ſeriei erit ille in quo numerus λ, vel $\lambda - 2$, vel $\lambda - 4$, &c. qui multiplicat angulum A, evadit æqualis 1. Ubi verò λ eſt numerus par, terminus ultimus ſeriei erit ille in quo numerus prædictus evadit æqualis 0, quo in caſu ſemiſſis tantùm ultimi termini ſumenda eſt; cùm enim ſeries hæc colligatur ex numero pari terminorum homologorum, quæ tamen, ubi λ eſt numerus par, conſtare debet ex terminorum numero impari, ideò duplum exhibet terminum ultimum.

Simili modo cùm ſit $2 \times \text{ſin.}\ A = \overline{l - m} \times \sqrt{-1}$, erit

$$2^{\lambda} \times \overline{\text{ſin.}\ A}|^{\lambda} = \sqrt{-1} \times \left\{ \begin{array}{l} l^{\lambda} - \lambda l^{\lambda-1} m + \lambda \times \frac{\lambda-1}{2} l^{\lambda-2} m^{2} \\ \quad - \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} l^{\lambda-3} m^{3} +, \\ \quad \text{\&c.} \\ \pm m^{\lambda} \mp \lambda m^{\lambda-1} l \pm \lambda \times \frac{\lambda-1}{2} m^{\lambda-2} l^{2} \\ \quad \mp \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} m^{\lambda-3} l \pm, \\ \quad \text{\&c.} \end{array} \right.$$

Terminis inferioribus hujus ſeriei præfiguntur alternatim ſigna + — ubi λ eſt numerus par, et ſigna — + ubi λ eſt numerus impar, adeoque in priore caſu eſt

$$2^{\lambda-1} \times \overline{\text{ſin.}\ A}|^{\lambda} = \overline{\sqrt{-1}}|^{\lambda} \text{ in } \frac{l^{\lambda} + m^{\lambda}}{2} - \lambda \times \frac{l^{\lambda-2} + m^{\lambda-2}}{2} + \lambda \times \frac{\lambda-1}{2} \times \frac{l^{\lambda-4} + m^{\lambda-4}}{2} - \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \times \frac{l^{\lambda-6} + m^{\lambda-6}}{2} +, \text{\&c.}$$

et in caſu poſteriori

$$2^{\lambda-1} \times \overline{\text{ſin.}\ A}|^{\lambda} = \overline{\sqrt{-1}}|^{\lambda} \text{ in } \frac{l^{\lambda} - m^{\lambda}}{2} - \lambda \times \frac{l^{\lambda-2} - m^{\lambda-2}}{2} + \lambda \times \frac{\lambda-1}{2} \times \frac{l^{\lambda-4} - m^{\lambda-4}}{2} - \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \times \frac{l^{\lambda-6} - m^{\lambda-6}}{2} +, \text{\&c.}$$

Adeoque ſi λ ſit numerus par, erit

$$\overline{\text{ſin.}\ A}|^{\lambda} = \frac{1}{2^{\lambda-1}} \text{ in } \pm \text{ coſ. } \lambda A \mp \lambda \text{ coſ. } \overline{\lambda - 2} \times A \pm \lambda \times \frac{\lambda-1}{2} \text{ coſ. } \overline{\lambda - 4} \times A \mp \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \text{ coſ. } \overline{\lambda - 6} \times A \pm, \text{\&c.}$$

Signa

Signa hîc alternatim mutantur, et ſuperiora ſunt adhibenda, ubi λ exprimit unum ex numeris 4, 8, 12, 16, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = 1$; inferiora autem adhibenda, ubi λ exprimit unum ex numeris 2, 6, 10, 14, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = -1$.

Si λ ſit numerus impar, cùm per lemma ſit $\frac{l^{\lambda} - m^{\lambda}}{2} \sqrt{-1} =$ ſin. λ A, et $\frac{l^{\lambda-2} - m^{\lambda-2}}{2} \sqrt{-1} =$ ſin. $\overline{\lambda - 2} \times A$, &c. habetur

$$\overline{\text{ſin. } A}|^{\lambda} = \frac{1}{2^{\lambda-1}} \text{ in } \pm \text{ſin. } \lambda A \mp \lambda \times \text{ſin. } \overline{\lambda - 2} \times A \pm \lambda \times \frac{\lambda - 1}{2} \text{ ſin. } \overline{\lambda - 4} \times A \mp \lambda \times \frac{\lambda - 1}{2} \times \frac{\lambda - 2}{3} \text{ ſin. } \overline{\lambda - 6} \times A \pm, \text{ \&c.}$$

ubi ſigna ſuperiora ſunt uſurpanda, cùm λ exprimit unum ex numeris 1, 5, 9, 13, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = \sqrt{-1}$; et ſigna inferiora, cùm λ fuerit unus ex numeris 3, 7, 11, 15, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = \sqrt{-1}$.

Notandum autem, ſeriei ultimum terminum eſſe illum in quo numerus λ, vel λ — 2, vel λ — 4, &c. eſt æqualis 1 ubi λ eſt numerus impar; atque terminum ultimum eſſe illum in quo prædictus numerus eſt æqualis 0 ubi λ eſt numerus par, quo in caſu ſemiſſis tantùm ultimi termini aſſumenda eſt ob rationem ſuperiùs datam.

Ex his ſinuum et coſinuum expreſſionibus alia hujuſmodi theoremata deducere liceret, ſed quæ hîc traduntur ad præſens inſtitutum ſufficiunt.

COROLL. IV.

Notum eſt fluentem fluxionis $\dot{A}$ coſ. A eſſe ſin. A, atque fluentem fluxionis $\dot{A}$ ſin. A eſſe ſin. verſ. A. Pariter ſi ſumatur arcus λA qui ſit ad arcum A ut numerus quilibet λ ad 1, cùm ſit $\lambda \dot{A}$ coſ. λA æqualis fluxioni ſinûs arcûs λA, erit flu. $\dot{A}$ coſ. $\lambda A = \frac{\text{ſin. } \lambda A}{\lambda}$, et flu. $\dot{A}$ ſin. $\lambda A = \frac{\text{ſin. verſ. } \lambda A}{\lambda}$. Itemque, ſi ad arcum λA adjungatur arcus datus d, cùm fluxio arcûs $\lambda A + d$ ſit æqualis $\lambda \dot{A}$, erit flu. $\dot{A}$ coſ. $\lambda A + d = \frac{\text{ſin. } \overline{\lambda A + d}}{\lambda}$, et flu. $\dot{A}$ ſin. $\overline{\lambda A + d} = \frac{\text{ſin. verſ. } \overline{\lambda A + d}}{\lambda}$.

Sumantur jam duo anguli, vel duo arcus λA et μA, qui ſint ad angulum, vel arcum A reſpectivè, ut λ et μ ad 1, atque per Coroll. II. habetur coſ. λA coſ. $\times \mu A = \frac{1}{2}$ coſ. $\overline{\lambda + \mu} \times A + \frac{1}{2}$ coſ. $\overline{\lambda - \mu} \times A$; unde erit fluens fluxionis $\dot{A}$ coſ. $\lambda A \times$ coſ. μA æqualis $\frac{\text{ſin. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}} + \frac{\text{ſin. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

Atque hoc methodo prodeunt ſequentes formulæ

1°. Flu. $\dot{A}$ coſ. $\lambda A \times$ coſ. $\mu A = \frac{\text{ſin. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}}$ $+ \frac{\text{ſin. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

2°. Flu. $\dot{A}$ ſin. $\lambda A \times$ ſin. $\mu A = - \frac{\text{ſin. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}}$ $+ \frac{\text{ſin. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

3°. Flu. $\dot{A}$ ſin. $\lambda A \times$ coſ. $\mu A = \dfrac{\text{ſin. verſ. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}} + \dfrac{\text{ſin. verſ. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

Advertendum autem eſt, ubi $\lambda = \mu$, tunc eſſe coſ. $\lambda A \times$ coſ. $\mu A = \frac{1}{2}$ coſ. $2 \lambda A + \frac{1}{2}$, ſin. $\lambda A \times$ ſin. $\mu A = - \frac{1}{2}$ coſ. $2 \lambda A + \frac{1}{2}$, ſin. $\lambda A \times$ coſ. $\mu A = \frac{1}{2}$ ſin. $2 \lambda A$; adeoque in hoc caſu formulæ præcedentes evadunt

1°. Flu. $\dot{A} \times \overline{\text{coſ. } \lambda A}|^2 = \dfrac{\text{ſin. } 2 \lambda A}{4 \lambda} + \dfrac{A}{2}$.

2°. Flu. $\dot{A} \times \overline{\text{ſin. } \lambda A}|^2 = - \dfrac{\text{ſin. } 2 \lambda A}{4 \lambda} + \dfrac{A}{2}$.

3°. Flu. $\dot{A} \times$ ſin. $\lambda A \times$ coſ. $\lambda A = \dfrac{\text{ſin. verſ. } 2 \lambda A}{4 \lambda}$.

Si angulo λA addatur angulus datus d, erit coſ. $\overline{\lambda A + d} \times$ coſ. $\mu A = \frac{1}{2}$ coſ. $\overline{\overline{\lambda + \mu} \times A + d} + \frac{1}{2}$ coſ. $\overline{\overline{\lambda - \mu} \times A + d}$, atque inde

1°. Flu. $\dot{A}$ coſ. $\overline{\lambda A + d} \times$ coſ. $\mu A = \dfrac{\text{ſin. } \overline{\overline{\lambda + \mu} \times A + d}}{2 \times \overline{\lambda + \mu}} + \dfrac{\text{ſin. } \overline{\overline{\lambda - \mu} \times A + d}}{2 \times \overline{\lambda - \mu}}$.

2°. Flu. $\dot{A}$ ſin. $\overline{\lambda A + d} \times$ ſin. $\mu A = - \dfrac{\text{ſin. } \overline{\overline{\lambda + \mu} \times A + d}}{2 \times \overline{\lambda + \mu}} + \dfrac{\text{ſin. } \overline{\overline{\lambda - \mu} \times A + d}}{2 \times \overline{\lambda - \mu}}$.

3°. Flu. $\dot{A}$ ſin. $\overline{\lambda A + d} \times$ coſ. $\mu A = \dfrac{\text{ſin. verſ. } \overline{\overline{\lambda + \mu} \times A + d}}{2 \times \overline{\lambda + \mu}} + \dfrac{\text{ſin. verſ. } \overline{\overline{\lambda - \mu} \times A + d}}{2 \times \overline{\lambda - \mu}}$.

4°. Flu. $\dot{A}$ coſ. $\overline{\lambda A+d}\times$ ſin. $\mu A = \dfrac{\text{ſin. verſ. }\overline{\overline{\lambda+\mu}\times A+d}}{2\times\overline{\lambda+\mu}}$ $- \dfrac{\text{ſin. verſ. }\overline{\overline{\lambda-\mu}\times A+d}}{2\times\overline{\lambda-\mu}}$.

Si fuerit $\lambda=\mu$, erit coſ. $\overline{\lambda A+d}\times$ coſ. $\lambda A =$ $\frac{1}{2}$ coſ. $\overline{2\lambda A+d}+\frac{1}{2}$ coſ. d, &c. adeoque formulæ præcedentes in has abeunt,

1°. Flu. $\dot{A}$ coſ. $\overline{\lambda A+d}\times$ coſ. $\lambda A = \dfrac{\text{ſin. }\overline{2\lambda A+d}}{4\lambda}$ $+\dfrac{\text{coſ. }d}{2}A$.

2°. Flu. $\dot{A}$ ſin. $\overline{\lambda A+d}\times$ ſin. $\lambda A = -\dfrac{\text{ſin. }\overline{2\lambda A+d}}{4\lambda}$ $+\dfrac{\text{coſ. }d}{2}A$.

3°. Flu. $\dot{A}$ ſin. $\overline{\lambda A+d}\times$ coſ. $\lambda A = \dfrac{\text{ſin. verſ. }\overline{2\lambda A+d}}{4\lambda}$ $+\dfrac{\text{ſin. }d}{2}A$.

4°. Flu. $\dot{A}$ coſ. $\overline{\lambda A+d}\times$ ſin. $\lambda A = \dfrac{\text{ſin. verſ. }\overline{2\lambda A+d}}{4\lambda}$ $-\dfrac{\text{ſin. }d}{2}A$.

PROPOSITIO I. PROBLEMA.

In ſyſtemate duorum planetarum circa Solem in orbibus penè circularibus revolventium, requiratur vis planetæ exterioris ad perturbandum motum interioris.

Revolvantur planetæ duo P et Q (Fig. 1.) in eodem plano circa Solem in S, et jungantur SP, SQ, PQ.

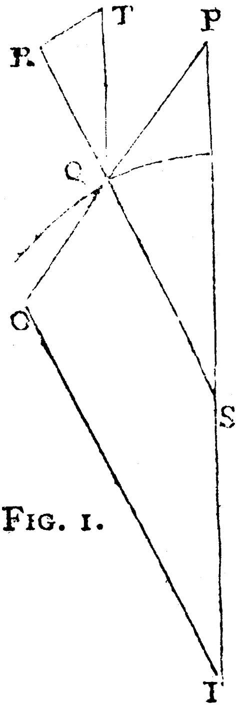

FIG. 1.

Orbis planetæ interioris Q, cujus motus hîc inveſtigamus, circularis ſupponitur niſi quatenùs mutatur ejus figura vi planetæ P; orbem verò planetæ P ut accuratè circularem habemus. Poſitâ ergò unitate pro diſtantiâ corporis Q à Sole ubi ambo planetæ verſantur in conjunctione cum Sole, fiant $SQ = x$, $PQ = z$, $SP = k$; tumque vis attractionis Solis in diſtantiâ æquali 1 ſit ad vim attractionis planetæ P in eâdem diſtantiâ ut 1 ad φ, eritque $\frac{\varphi}{z^2}$ gravitas planetæ Q in planetam P. Producatur jam, ſi opus eſt, PQ ad O ut ſit $PO = \frac{\varphi}{z^2}$, et ductâ OI parallelâ ipſi QS occurrente rectæ PS productæ in I, propter triangula ſimilia PQS, POI, erit $PQ \cdot PS :: PO \cdot PI$, hoc eſt, $PI = \frac{\varphi k}{z^3}$, atque $PQ \cdot QS :: PO \cdot OI$, hoc eſt, $OI = \frac{\varphi x}{z^3}$. Sed, quia parùm differt x ab unitate et admodùm exigua eſt vis φ, pro x ſcribi poteſt 1 in omnibus iis terminis qui ducuntur in φ, adeoque $OI = \frac{\varphi}{z^3}$. Ex vi PI auferatur vis $\frac{\varphi}{k^2}$ qua gravitat Sol in planetam P, et vis reſidua $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$ eſt ea qua perturbatur motus planetæ Q in directione parallelâ rectæ PS: nàm cùm motus planetarum referantur ad Solem ſpectatum tanquàm immotum, vis $\frac{\varphi k}{z^3}$ pars ea $\frac{\varphi}{k^2}$, qua ſimul urgentur Sol et planeta Q verſus P ſecundum lineas parallelas, non mutat corporum S et Q ſitum ad ſe invicem, ideoque differentia virium ſola perturbationem inducit.

Quare differentia illa, nimirùm $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$, exponatur per lineam QT parallelam rectæ PS, et in SQ demiſſo perpendiculo TR, vis QT reſolvetur in vires TR, QR, eritque vis QT ad vim TR ut radius 1 ad ſinum anguli QSP, adeoque vis TR $= \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. QSP, et vis QR $= \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ coſ. QSP. Ex vi autem QR tollatur vis OI utpotè in contrarium agens, et manebit vis $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ coſ. QSP $- \frac{\varphi}{z^3}$. Vires igitur, quibus planeta P perturbat motum planetæ Q quatenùs in eodem plano moventur, ſunt

1°. Vis TR ad radium QS perpendicularis, qua augetur vel minuitur area tempore dato deſcripta, eſtque æqualis $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. QSP.

2°. Vis $\frac{\varphi}{z^3} \times \overline{k \text{ coſ. QSP} - 1} - \frac{\varphi}{k^2}$ coſ. QSP, qua retrahitur planeta Q à Sole in directione radii SQ.

Ut autem harumce virium expreſſiones formam induant calculo accommodam, ope trianguli PSQ habebitur $\overline{PQ}^2 = zz = kk + xx - 2kx \times$ coſ. QSP, ſive, poſitâ $x = 1$ ob rationem dictam, $zz = kk$

$+ 1 - 2k \times \text{coſ.}\ QSP$. Aſſumatur jam angulus s qui ſemper ſit ad angulum QSP in ratione n ad 1, eritque $QSP = \frac{1}{n}s$, et poſito $kk + 1 = tt$, et $\frac{2k}{t^2} = b$, erit $z^2 = t^2 \times \overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}$, hincque $\frac{1}{z^3} = \frac{1}{t^3} \times \overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}|^{-\frac{3}{2}}$. Si b fuerit unitati ferè æqualis, et evolvatur quantitas $\overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}|^{-\frac{3}{2}}$ in ſeriem modo ſolito, ſeries illa parùm convergit, eſtque ad operationes analyticas minùs commoda. Series igitur alia inveſtiganda eſt, et quia ex lemmate patet hujuſmodi quantitatem $\overline{\text{coſ.}\ A}|^{k}$ exprimi poſſe aggregato terminorum, quorum ſinguli ducuntur in coſinus angulorum qui ſunt anguli A multiplices, generatim ſupponemus $\overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}|^{m} = R + S\ \text{coſ.}\ \frac{1}{n}s + T\ \text{coſ.}\ \frac{2}{n}s + V\ \text{coſ.}\ \frac{3}{n}s + W\ \text{coſ.}\ \frac{4}{n}s +$, &c.

Atque ut inveniantur valores coefficientium R, S, T, &c. ſumatur utrinque fluxio, nempe $\frac{mb}{n}\dot{s} \times \text{ſin.}\ \frac{1}{n}s \times \overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}|^{m-1} = - S \times \frac{1}{n}\dot{s} \times \text{ſin.}\ \frac{1}{n}s - T \times \frac{2}{n}\dot{s} \times \text{ſin.}\ \frac{2}{n}s - V \times \frac{3}{n}\dot{s} \times \text{ſin.}\ \frac{3}{n}s - W \times \frac{4}{n}\dot{s} \times \text{ſin.}\ \frac{4}{n}s -$, &c. atque ducatur æquatio hæc in $\overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}$, et ſubſtituto pro $\overline{1 - b\ \text{coſ.}\ \frac{1}{n}s}|^{m}$ ipſius valore $R + S\ \text{coſ.}\ \frac{1}{n}s + T\ \text{coſ.}\ \frac{2}{n}s +$, &c. fiet $mb \times \text{ſin.}\ \frac{1}{n}s$

$\times$

$$\times\overline{\mathrm{R}+\mathrm{S}\,\text{coſ.}\,\tfrac{1}{n}s+\mathrm{T}\,\text{coſ.}\,\tfrac{2}{n}s+\mathrm{V}\,\text{coſ.}\,\tfrac{3}{n}s+,\&c.=1-b\,\text{coſ.}\,\tfrac{1}{n}s}$$

$$\times\overline{-\mathrm{S}\times\text{ſin.}\,\tfrac{1}{n}s+2\mathrm{T}\times\text{ſin.}\,\tfrac{2}{n}s\times3\mathrm{V}\times\text{ſin.}\,\tfrac{3}{n}s+\mathrm{W}\times\text{ſin.}\,\tfrac{4}{n}s+,\&c.}$$

et factâ multiplicatione, cùm ſit (per Coroll. I. Lem.) $\text{ſin.}\,\frac{1}{n}s\times\text{coſ.}\,\frac{r}{n}s=\frac{1}{2}\,\text{ſin.}\,\frac{r+1}{n}s-\frac{1}{2}\,\text{ſin.}\,\frac{r-1}{n}s$, ac $\text{ſin.}\,\frac{r}{n}s\times\text{coſ.}\,\frac{1}{n}s=\frac{1}{2}\,\text{ſin.}\,\frac{r+1}{n}s+\frac{1}{2}\,\text{ſin.}\,\frac{r-1}{n}s$, emerget

$$\left.\begin{array}{l}+2mb\mathrm{R}\\+2\mathrm{S}\\-2b\mathrm{T}\\-mb\mathrm{T}\end{array}\right\}\times\text{ſin.}\,\tfrac{1}{n}s\left\{\begin{array}{l}+mb\mathrm{S}\\-b\mathrm{S}\\+4\mathrm{T}\\-3b\mathrm{V}\\-mb\mathrm{V}\end{array}\right\}\times\text{ſin.}\,\tfrac{2}{n}s\left\{\begin{array}{l}+mb\mathrm{T}\\-2b\mathrm{T}\\+6\mathrm{V}\\-4b\mathrm{W}\\-mb\mathrm{W}\end{array}\right\}\times\text{ſin.}\,\tfrac{3}{n}s,\ \&c.=0$$

Deinde nihilo æquando ſingulos terminos, prodeunt $\mathrm{T}=\frac{2\mathrm{S}+2mb\mathrm{R}}{\overline{m+2}\times b}$, $\mathrm{V}=\frac{4\mathrm{T}+\overline{m-1}\times b\mathrm{S}}{\overline{m+3}\times b}$, $\mathrm{W}=\frac{6\mathrm{V}+\overline{m-2}\times b\mathrm{T}}{\overline{m+4}\times b}$, &c. quorum valorum progreſſus ſatis manifeſtus eſt.

Datis igitur primis duobus coefficientibus R et S, dabuntur et reliqui: R et S autem ſic inveniuntur.

Eſt $\overline{1-b\,\text{coſ.}\,\frac{1}{n}s}\,\Big|^{m}=1-mb\,\text{coſ.}\,\frac{1}{n}s+m\times\frac{m-1}{2}b^2\,\overline{\text{coſ.}\,\frac{1}{n}s}\,\Big|^{2}-m\times\frac{m-1}{2}\times\frac{m-2}{3}b^3\,\overline{\text{coſ.}\,\frac{1}{n}s}\,\Big|^{3}$, &c. $=\mathrm{R}+\mathrm{S}\,\text{coſ.}\,\frac{1}{n}s+\mathrm{T}\,\text{coſ.}\,\frac{2}{n}s+\mathrm{V}\,\text{coſ.}\,\frac{3}{n}s+$, &c. Evolvantur termini $\overline{\text{coſ.}\,\frac{1}{n}s}\,\Big|^{2}$, $\overline{\text{coſ.}\,\frac{1}{n}s}\,\Big|^{4}$, $\overline{\text{coſ.}\,\frac{1}{n}s}\,\Big|^{6}$, &c. per methodum traditam in Coroll. III. Lem. ac, collectis ſimul omnibus terminis qui nullo coſinu afficiuntur, prodibit

$$R = 1 + \frac{m}{2} \times \frac{m-1}{2} b^2 + \frac{m}{2} \times \frac{m-1}{2} \times \frac{m-2}{4} \times \frac{m-3}{4} b^4$$
$$+ \frac{m}{2} \times \frac{m-1}{2} \times \frac{m-2}{4} \times \frac{m-3}{4} \times \frac{m-4}{6} \times \frac{m-5}{6} b^6 +, \text{ \&c.}$$
cujus ſeriei progreſſio ſatis patet; atque adeò, cum ſit in hoc noſtro problemate $m = -\frac{3}{2}$, erit
$$R = 1 + \frac{3\times5}{4\times4} b^2 + \frac{3\times5}{4\times4} \times \frac{7\times9}{8\times8} b^4 + \frac{3\times5}{4\times4} \times \frac{7\times9}{8\times8}$$
$$\times \frac{11\times13}{12\times12} b^6 + \frac{3\times5}{4\times4} \times \frac{7\times9}{8\times8} \times \frac{11\times13}{12\times12} \times \frac{15\times17}{16\times16} b^8 +, \text{ \&c.}$$

Inſpicienti indolem hujus ſeriei patebit terminum quemlibet æquari termino antecedenti ducto in $\frac{\overline{r+1}\times\overline{r-1}}{r^2} b^2$, ſive $\frac{r^2-1}{r^2} b^2$, r exiſtente æquali numero quadruplicato terminorum præcedentium: ſic terminus ſextus, quia habetur in hoc caſu $r = 5 \times 4 = 20$, æqualis eſt termino quinto $\frac{3\times5}{4\times4} \ldots\ldots \frac{15\times17}{16\times16} b^8$ ducto in $\frac{19\times21}{20\times20} b^2$.

Termino igitur quovis hujus ſeriei dicto B, terminus ſubſequens erit $Bb^2 \times \frac{r^2-1}{r^2}$: et manente deinceps eodem, quem in hoc termino habet, numeri r valore, termini ſubſequentes erunt, $Bb^4 \times \frac{r^2-1}{r^2} \times \frac{\overline{r+4}|^2-1}{\overline{r+4}|^2}$, $Bb^6 \times \frac{r^2-1}{r^2} \times \frac{\overline{r+4}|^2-1}{\overline{r+4}|^2} \times \frac{\overline{r+8}|^2-1}{\overline{r+8}|^2}$, $Bb^8 \times \frac{r^2-1}{r^2} \ldots\ldots \frac{\overline{r+12}|^2-1}{\overline{r+12}|^2}$, &c. Sed eſt $\frac{r^2-1}{r^2} = 1 - \frac{1}{r^2}$, $\frac{\overline{r+4}|^2-1}{\overline{r+4}|^2} = 1 - \frac{1}{\overline{r+4}|^2}$, &c. et ſi fuerit r

numerus aliquantùm magnus, erit $\frac{r^2-1}{r^2}\times\frac{\overline{r+4}|^2-1}{\overline{r+4}|^2}$ $=1-\frac{1}{r^2}-\frac{1}{\overline{r+4}|^2}$, et $\frac{r^2-1}{r^2}\times\frac{\overline{r+4}|^2-1}{\overline{r+4}|^2}\times\frac{\overline{r+8}|^2-1}{\overline{r+8}|^2}$ $=1-\frac{1}{r^2}-\frac{1}{\overline{r+4}|^2}-\frac{1}{\overline{r+8}|^2}$, atque ita porrò, rejiciendo fractiones hujus generis $\frac{1}{r^2\times\overline{r+4}|^2}$ et alias his minores.

Unde termini omnes prædicti, incipiendo à termino B, erunt

$$\begin{array}{rcl}
B+Bb^2+Bb^4+Bb^6+Bb^8+,\ \&c. & = & B\times\frac{1}{1-b^2}\\
-\frac{Bb^2}{r^2}-\frac{Bb^4}{r^2}-\frac{Bb^6}{r^2}-\frac{Bb^8}{r^2}-,\ \&c. & = & -\frac{B}{r^2}\times\frac{b^2}{1-b^2}\\
-\frac{Bb^4}{\overline{r+4}|^2}-\frac{Bb^6}{\overline{r+4}|^2}-\frac{Bb^8}{\overline{r+4}|^2}-,\ \&c. & = & -\frac{B}{\overline{r+4}|^2}\times\frac{b^4}{1-b^2}\\
-\frac{Bb^6}{\overline{r+8}|^2}-\frac{Bb^8}{\overline{r+8}|^2}-,\ \&c. & = & -\frac{B}{\overline{r+8}|^2}\times\frac{b^6}{1-b^2}\\
-\frac{Bb^8}{\overline{r+12}|^2}-,\ \&c. & = & -\frac{B}{\overline{r+12}|^2}\times\frac{b^8}{1-b^2}\\
\&c. & & \&c.
\end{array}$$

ac proinde tandem fit

$$R=1+\frac{3\times5}{4\times4}b^2+\frac{3\times5}{4\times4}\times\frac{7\times9}{8\times8}b^4+\frac{3\times5}{4\times4}\times\frac{7\times9}{8\times8}\times\frac{11\times13}{12\times12}b^6\times\frac{3\times5}{4\times4}\ldots\ldots\frac{15\times17}{16\times16}b^8+,\ \&c.+\frac{B}{1-b^2}\times\overline{1-\frac{b^2}{r^2}-\frac{b^4}{\overline{r+4}|^2}-\frac{b^6}{\overline{r+8}|^2}-\frac{b^8}{\overline{r+12}|^2}-\frac{b^{10}}{\overline{r+16}|^2}-,\ \&c.}$$

Unde ſi, computatis, exempli gratiâ, decem terminis, undecimus deſignetur per B, erit $r=10\times4=40$, et ſumma illorum decem terminorum addita ſummæ ſeriei $\frac{B}{1-b^2}\times\overline{1-\frac{b^2}{r^2}-\frac{b^4}{\overline{r\times4}|^2}-}$, &c. dabit valorem ipſius R.

Simili modo ſi in æquatione prædictâ $1-mb\,\text{coſ.}\,\frac{1}{n}s+m\times\frac{m-1}{2}b^2\times\overline{\text{coſ.}\,\frac{1}{n}s}|^2+m\times\frac{m-1}{2}\times\frac{m-2}{3}b^3\times\overline{\text{coſ.}\,\frac{1}{n}s}|^3+$, &c. $=R+S\,\text{coſ.}\,\frac{1}{n}s+T\,\text{coſ.}\,\frac{2}{n}s+V\,\text{coſ.}\,\frac{3}{n}s+$, &c. evolvantur quantitates $\overline{\text{coſ.}\,\frac{1}{n}s}|^3$. $\overline{\text{coſ.}\,\frac{1}{n}s}|^5$, $\overline{\text{coſ.}\,\frac{1}{n}s}|^7$, &c. in ſuos valores, prout in Coroll. III. Lem. edoctum eſt, et colligantur omnes termini qui ducuntur in $\text{coſ.}\,\frac{1}{n}s$ exurget

$$S=-mb-m\times\frac{m-1}{2}\times\frac{m-2}{4}b^3-m\times\frac{m-1}{2}\times\frac{m-2}{4}\times\frac{m-3}{4}\times\frac{m-4}{6}b^5-m\times\frac{m-1}{2}\times\frac{m-2}{4}\times\frac{m-3}{4}\times\frac{m-4}{6}\times\frac{m-5}{6}\times\frac{m-6}{8}b^7-\text{, \&c.}$$

ſive, poſito $m=-\frac{3}{2}$,

$$S=\tfrac{3}{2}b+\tfrac{3}{2}\times\frac{5\times7}{4\times8}b^3+\tfrac{3}{2}\times\frac{5\times7}{4\times8}\times\frac{9\times11}{8\times12}b^5+\tfrac{3}{2}\times\frac{5\times7}{4\times8}\times\frac{9\times11}{8\times12}\times\frac{13\times15}{12\times16}b^7+\tfrac{3}{2}\cdots\cdots\frac{13\times15}{12\times16}\times\frac{17\times19}{16\times20}b^9+\text{, \&c.}$$

Patet autem terminum quemlibet hujus ſeriei æquari termino antecedenti ducto in $\frac{\overline{r+1}\times\overline{r+3}}{r\times\overline{r+4}}b^2$, exiſtente r æquali numero terminorum præcedentium quadruplicato: ſic terminus ſextus, quia tunc $r=5\times4=20$, eſt æqualis termino quinto $\frac{3}{2}\cdots\cdots\frac{17\times19}{16\times20}b^8$ ducto in $\frac{21\times23}{20\times24}b^2$. Quamobrem termino quovis hujus ſeriei dicto B, terminus ſubſequens erit $Bb^2\times\frac{\overline{r+1}\times\overline{r+3}}{r\times\overline{r+4}}$, ſive $Bb^2\times\overline{1+\frac{3}{r\times\overline{r+4}}}$, et manente jam eodem valore numeri r, termini reliqui erunt, $Bb^4\times\overline{1+\frac{3}{r\times\overline{r+4}}}\times\overline{1+\frac{3}{\overline{r+4}\times\overline{r+8}}}$, $Bb^6\times\overline{1+\frac{3}{r\times\overline{r+4}}}\times\overline{1+\frac{3}{\overline{r+4}\times\overline{r+8}}}\times\overline{1+\frac{3}{\overline{r+8}\times\overline{r+12}}}$, &c. Sed ſi fuerit r numerus aliquantùm magnus, erit $\overline{1+\frac{3}{r\times\overline{r+4}}}\times\overline{1+\frac{3}{\overline{r+4}\times\overline{r+8}}}=1+\frac{3}{r\times\overline{r+4}}+\frac{3}{\overline{r+4}\times\overline{r+8}}$ quamproximè, et $\overline{1+\frac{3}{r\times\overline{r+4}}}\times\overline{1+\frac{3}{\overline{r+4}\times\overline{r+8}}}\times\overline{1+\frac{3}{\overline{r+8}\times\overline{r+12}}}=1+\frac{3}{r\times\overline{r+4}}+\frac{3}{\overline{r+4}\times\overline{r+8}}+\frac{3}{\overline{r+8}\times\overline{r+12}}$, &c. Unde termini omnes prædicti incipientes à termino B erunt

$$B+Bb^2+Bb^4+Bb^6+Bb^8+\text{, \&c.}=\frac{B}{1-b^2}$$

$$+\frac{3Bb^2}{r\times\overline{r+4}}+\frac{3Bb^4}{r\times\overline{r+4}}+\frac{3Bb^6}{r\times\overline{r+4}}+\frac{3Bb^8}{r\times\overline{r+4}}+\text{, \&c.}=\frac{3B}{r\times\overline{r+4}}\times\frac{b^2}{1-b^2}$$

$$+\frac{3Bb^4}{\overline{r+4}\times\overline{r+8}}+\frac{3Bb^6}{\overline{r+4}\times\overline{r+8}}+\frac{3Bb^8}{\overline{r+4}\times\overline{r+8}}+\text{, \&c.}=\frac{3B}{\overline{r+4}\times\overline{r+8}}\times\frac{b^4}{1-b^2}$$

$$+\frac{3Bb^6}{\overline{r+8}\times\overline{r+12}}+\frac{3Bb^8}{\overline{r+8}\times\overline{r+12}}+\text{, \&c.}=\frac{3B}{\overline{r+8}\times\overline{r+12}}\times\frac{b^6}{1-b^2}$$

$$+\frac{3Bb^8}{\overline{r+12}\times\overline{r+16}}+\text{, \&c.}=\frac{3B}{\overline{r+12}\times\overline{r+16}}\times\frac{b^8}{1-b^2}$$

&c. &c.

Ac proinde erit

$$S = \tfrac{3}{2}b + \tfrac{3}{2}\times\frac{5\times7}{4\times8}b^3 + \tfrac{3}{2}\times\frac{5\times7}{4\times8}\times\frac{9\times11}{8\times12}b^5 + \tfrac{3}{2}\times\frac{5\times7}{4\times8}\times\frac{9\times11}{8\times12}\times\frac{13\times15}{12\times16}b^7 +, \&c. + \frac{B}{1-b^2}$$
$$\times\overline{1+\frac{3b^2}{r\times\overline{r+4}}+\frac{3b^4}{\overline{r+4}\times\overline{r+8}}+\frac{3b^6}{\overline{r+8}\times\overline{r+12}}+\frac{3b}{\overline{r+12}\times\overline{r+16}}+,\&c.}$$

Itaque ſi, computatis, exempli gratiâ, quindecim terminis, decimus ſextus deſignetur per B, erit $r = 15\times4 = 60$, et ſumma terminorum quindecim illorum addita ſummæ ſeriei $\frac{B}{1-b^2}\times\overline{1+\frac{3b^2}{r\times\overline{r+4}}+\frac{3b^4}{\overline{r+4}\times\overline{r+8}}+,\ \&c.}$ dabit valorem coefficientis S.

Determinatis hoc pacto quantitatibus aſſumptis R, S, T, &c. jam ut ad expreſſiones virium revertamur, vis TR ad radium QS perpendicularis erat $\overline{\frac{\varphi k}{z^3}-\frac{\varphi}{k^2}}$ $\times$ ſin. QSP; ſed poſuimus angulum QSP $= \frac{1}{n}s$, eſtque $\frac{1}{z^3} = \frac{1}{t^3}$ in R $+$ S coſ. $\frac{1}{n}s$ $+$ T coſ. $\frac{2}{n}s$ $+$ V coſ. $\frac{3}{n}s$ $+$ W coſ. $\frac{4}{n}s$ $+$, &c.

Unde vis TR $= \frac{\varphi k}{t^3}$ in $\overline{R-\frac{t^3}{k^3}-\frac{T}{2}}$ $\times$ ſin. $\frac{1}{n}s$ $+\frac{S-V}{2}$ ſin. $\frac{2}{n}s$ $+\frac{T-W}{2}$ ſin. $\frac{3}{n}s$ $+\frac{V-X}{2}$ ſin. $\frac{4}{n}s$ $+$, &c.

Et vis quæ planetam Q diſtrahit à Sole in directione radii QS erat $\frac{\varphi}{z^3}\times$ $\overline{k\ \text{coſ. QSP}-1}$ $-\frac{\varphi}{t^3}$ coſ. QSP, hoc eſt, $\frac{\varphi}{t^3}$ in $\frac{kS}{2}-R+\overline{kR+\frac{kT}{2}-\frac{t^3}{k^2}-S}$

$\times$ coſ.

$$\times \text{coſ.}\, \frac{1}{n}s + \frac{kS+kV-2T}{2} \times \text{coſ.}\, \frac{2}{n}s + \frac{kT+kW-2V}{2}\, \text{coſ.}\, \frac{3}{n}s + \frac{kV+kX-2W}{3}\, \text{coſ.}\, \frac{4}{n}s +,\ \&c.$$ Q. E. I.

PROPOSITIO II. PROBLEMA.

Inæqualitates motûs planetæ interioris ex viribus prædictis ortas inveſtigare.

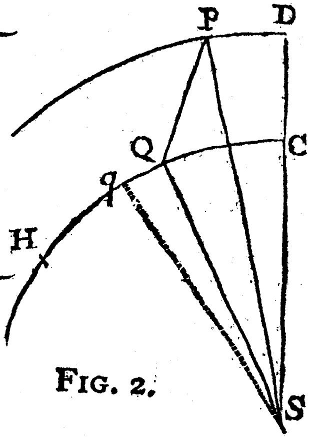

FIG. 2.

Exeant ſimul planetæ P, Q (Fig. 2.) de locis D, C, ubi jacebant in eâdem rectâ cum Sole poſito in S, et poſt aliquod temporis ſpatium reperiantur in P et Q, et jungantur SP, SQ, PQ. Eſto CS $= 1$, et arcus circularis CQ ſive angulus CSQ $= s$; denotent præthereà P et Q reſpectivè tempora periodica planetarum P et Q, eritque ang. QSC : ang. PSD :: P : Q, adeoque angulus QSP : ang. QSC :: P — Q : P, unde ang. QSP $= \frac{1}{n}s$, poſito $n = \frac{P}{P-Q}$. Vis attractionis Solis ad diſtantiam QS, et tempus quo corpus, eâdem vi uniformiter agente, impulſum acquirere poſſet eam velocitatem, qua planeta Q in circulo CQ revolvitur, tùm illa ipſa velocitas, exponantur ſigillatim per unitatem; et ſi, ſumpto arcu CH $=$ CS $= 1$, CH exprimat tempus illud unitati æquale, arcus quilibet quàm minimus Qq exprimet tempus quo uniformi illâ velocitate deſcribitur.

Unde, cum velocitates viribus quibuſvis conſtantibus genitæ ſint ut ipſæ vires et tempora, quibus hæ velocitates generantur, conjunctim; erit velocitas 1 planetæ Q in circulo CQ revolventis ad incrementum vel decrementum velocitatis vi Z genitum (ſcripto nempe Z pro vi planetæ P normaliter ad radium QS agente, prout eſt in propoſitione præcedente definita) quo tempore planeta Q deſcribit arcum quàm minimum Qq, ut vis attractionis Solis 1 ducta in tempus CH ſive 1, ad vim Z ductam in tempus deſcriptionis arcûs Qq ſive in ipſum in arcum Qq: adeoque incrementum vel decrementum velocitatis vi Z genitum, quo tempore deſcribitur arcus Qq, exprimetur per $Z \times Qq$ ſive $Z \times \dot{s}$.

Eſt autem $Z = \frac{\varphi k}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{ſin.}\, \frac{1}{n} s + \frac{S - V}{2} \text{ſin.}\, \frac{2}{n} s + \frac{T - W}{2} \text{ſin.}\, \frac{3}{n} s +$, &c. et hac quantitate ductâ in $\dot{s}$, tùm ſumptâ fluente, prodit velocitatis accceleratio ſive retardatio, quam voco U, genita quo tempore deſcribitur à planeta Q arcus CQ, æqualis $\frac{\varphi k n}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{ſin. verſ.}\, \frac{1}{n} s + \frac{S - V}{4} \text{ſin. verſ.}\, \frac{2}{n} s + \frac{T - W}{6} \text{ſin. verſ.}\, \frac{3}{n} s + \frac{V - X}{8} \text{ſin. verſ.}\, \frac{4}{n} s +$, &c. ſive poſito $b = R - \frac{t^3}{k^3} - \frac{T}{2} + \frac{S - V}{4} + \frac{T - W}{6} + \frac{V - X}{8} +$, &c. $U = \frac{\varphi k n}{t^3}$ in $b - \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{coſ.}\, \frac{1}{n} s - \frac{S - V}{4} \text{coſ.}\, \frac{2}{n} s - \frac{T - W}{6} \text{coſ.}\, \frac{3}{n} s - \frac{V - X}{8} \text{coſ.}\, \frac{4}{n} s -$, &c.

Hoc pacto obtinetur variatio velocitatis in hypotheſi quòd revolvatur planeta Q ſemper ad eamdem diſtantiam à Sole, quod in præcedenti calculo ſupponi poteſt, cùm tantillùm varietur diſtantia S Q actione planetæ P.

Hoc facto, ut inveſtigetur variatio diſtantiæ planetæ Q à Sole, fingamus planetam deſcripſiſſe, non arcum circularem C Q, ſed arcum curvæ C r (Fig. 3.) et reperiri in puncto r ubi radius S Q productus ſecat curvam.

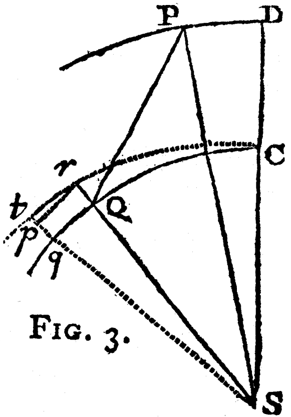

FIG. 3.

Ducatur recta S t viciniſſima ipſi S Q occurrens circulo et curvæ q et t; tùm centro S et radio S r deſcribatur arcus rp, ſitque $Sr = x$. Si planeta Q urgeretur ſolâ vi tendente ad centrum S, deſcriberet areas temporibus proportionales, atque adeò, cùm ipſius velocitas angularis in loco C ſupponatur eſſe 1, in loco r foret æqualis $\frac{1}{x}$; ſed in illo quem exhibet ſchema ſitu planetarum minuitur hæc velocitas quantitate U ſuprà definitâ, unde velocitas angularis in loco r erit $\frac{1}{x} - U$; et tempus, quo deſcriberetur arcus Q q velocitate 1, eſt ad tempus quo deſcribitur arcus rp velocitate $\frac{1}{x} - U$, ut Q q ad $\frac{rp}{\frac{1}{x} - U}$, hoc eſt, ut $\dot{s}$ ad $\frac{x\dot{s}}{\frac{1}{x} - U}$; unde, cùm $\dot{s}$ exprimat ex jam dictis tempus deſcriptionis arcûs Q q velocitate 1, exprimet quantitas $\frac{x\dot{s}}{\frac{1}{x}-U}$ tempus quo deſcribitur arcus rp velocitate $\frac{1}{x}-U$. His poſitis, quoniam planetæ Q receſſus à centro vel ad idem acceſſus pendet ex differentiâ virium, centrifugæ ſcilicet et centripetæ, quibus urgetur in Q; ſi hæc differentia virium dicatur P, et v denotet velocitatem aſcenſûs vel deſcenſûs planetæ Q ſecundum radium SQ, per idem planè ratiocinium, quod mox uſurpavimus in inveſtigatione velocitatis U, habebitur $\dot{v} = P \times \frac{x\dot{s}}{\frac{1}{x}-U}$.

Quoniam ex hypotheſi planeta Q, ſepoſitâ actione planetæ P, deſcriberet circulum, vires (centripeta et centrifuga) ſibi invicem et unitati forent æquales: exiſtente autem planetâ Q in r, ipſius attractio in Solem eſt $\frac{1}{x^2}$, ex qua auferenda eſt vis ea qua juxta propoſitionem præcedentem diſtrahitur à Sole, nimirùm $\frac{\varphi}{t^3}$ in $A + B$ coſ. $\frac{1}{n}s + C$ coſ. $\frac{2}{n}s + D$ coſ. $\frac{3}{n}s + E$ coſ. $\frac{4}{n}s +$, &c. poſitis $A = \frac{kS}{2} - R$, $B = kR + \frac{kT}{2} - \frac{t^3}{k^2} - S$, $C = \frac{kS+kV-2T}{2}$, $D = \frac{kT+kW-2V}{2}$, $E = \frac{kV+kX-2W}{2}$, &c. atque harum virium differentia componit vim centripetam.

Vis autem centrifuga eſt ſemper in ratione duplicatâ areæ temporis momento deſcriptæ directè et triplicatâ diſtantiæ inverſè; unde ſi hæc vis fuerit æqualis

1,

1, ubi incepit planeta movere in C, erit æqualis $x^2 \times \overline{\frac{1}{x} - U}\big|^2 \times \frac{1}{x^3} = \frac{1}{x} \times \overline{\frac{1}{x} - U}\big|^2$ ubi movetur in r. Differentia igitur inter vim centrifugam et centripetam, qua urgetur planeta in r ſupra deſignata per P, eſt

$$\overline{\frac{1}{x} \times \overline{\frac{1}{x} - U}\big|^2 - \frac{1}{x^2} + \frac{\phi}{t^3}}$$

$$\times A + B\,\text{coſ.}\,\frac{1}{n}s + C\,\text{coſ.}\,\frac{2}{n}s + D\,\text{coſ.}\,\frac{3}{n}s +, \text{\&c.}$$

hincque habetur $\dot{v} = \dot{s} \times \overline{\frac{1}{x} - U} - \frac{\dot{s}}{x \times \frac{1}{x} - U} + \frac{\phi}{t^3}$

$$\times \frac{x\dot{s}}{\frac{1}{x} - U} \times \overline{A + B\,\text{coſ.}\,\frac{1}{n}s + C\,\text{coſ.}\,\frac{2}{n}s +, \text{\&c.}}$$

Vires, quibus perturbatur motus planetæ Q, cùm exprimantur ſeriebus quorum termini ducuntur in ſinum vel coſinum anguli $\frac{1}{n}s$, vel anguli hujus multiplicis, fingemus differentiam inter diſtantias SQ et Sr exprimi ſerie ſimili, ac proptereà ponemus $x = 1 - Q + K\,\text{coſ.}\,\frac{1}{n}s + L\,\text{coſ.}\,\frac{2}{n}s + M\,\text{coſ.}\,\frac{3}{n}s + N\,\text{coſ.}\,\frac{4}{n}s$, &c. exiſtente $Q = K + L + M + N +$, &c. ut ſit Sr, ſive $x = 1$, ubi planetæ Q et P incipiunt movere à lineâ conjunctionis SCD. Quantitates autem aſſumptæ K, L, M, &c. ſunt exiguæ, ideoque erit $\frac{1}{x} = 1 + Q - K\,\text{coſ.}\,\frac{1}{n}s - L\,\text{coſ.}\,\frac{2}{n}s - M\,\text{coſ.}\,\frac{3}{n}s - N\,\text{coſ.}\,\frac{4}{n}s -$, &c. quamproximè.

proximè. Subſtituantur ergò in æquatione ſuprà traditâ valores quantitatum x, $\frac{1}{x}$, et U; et ſumptâ fluente, rejectis iis terminis qui ducuntur in altiorem quàm unam dimenſionem quantitatum φ, Q, K, L, &c. prodit $v = -\overline{\frac{2\varphi khn}{t^3} - \frac{\varphi}{t^3}A - Q} \times s$

$$+\overline{\frac{2\varphi kn}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} + \frac{\varphi}{t^3}B - K} \times n \times \text{ſin.}\, \frac{1}{n}s$$

$$+\overline{\frac{\varphi kn}{t^3} \times \frac{S - V}{4} + \frac{\varphi}{t^3} \times \frac{C}{2} - \frac{L}{2}} \times n \times \text{ſin.}\, \frac{2}{n}s$$

$$+\overline{\frac{\varphi kn}{t^3} \times \frac{T - W}{9} + \frac{\varphi}{t^3} \times \frac{D}{3} - \frac{M}{3}} \times n \times \text{ſin.}\, \frac{3}{n}s$$

$$+\overline{\frac{\varphi kn}{t^3} \times \frac{V - X}{16} + \frac{\varphi}{t^3} \times \frac{E}{4} - \frac{N}{4}} \times n \times \text{ſin.}\, \frac{4}{n}s +, \text{ \&c.}$$

$+$ Z, deſignante Z quantitatem idoneam qua compleatur fluens. At, quoniam velocitas v ſupponitur nulla evadere, non ſolum ubi s, ſive arcus CQ $= 0$, id eſt, ubi planetæ verſantur in primâ illâ conjunctione, ſed etiam in omnibus aliis conjunctionibus ſubſequentibus, hoc eſt, ubi eſt angulus $\frac{1}{n}s$, ſeu PSQ $= 0$, vel $= r \times 180°$, ſcripto ſcilicet r pro quovis ex numeris naturalibus 1, 2, 3, 4, &c. fiet Z $= \overline{\frac{2\varphi khn}{t^3} - \frac{\varphi}{t^3}A - Q} \times s$ adeoque

$$v = \overline{\frac{2\varphi kn}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} + \frac{\varphi}{t^3}B - K} \times n \times \text{ſin.}\, \frac{1}{n}s$$

$$+\overline{\frac{\varphi kn}{t^3} \times \frac{S - V}{4} + \frac{\varphi}{t^3} \times \frac{C}{2} - \frac{L}{2}} \times n \times \text{ſin.}\, \frac{2}{n}s$$

$$+\overline{\frac{\varphi kn}{t^3} \times \frac{T - W}{9} + \frac{\varphi}{t^3} \times \frac{D}{3} - \frac{M}{3}} \times n \times \text{ſin.}\, \frac{3}{n}s$$

$+$

$$+\overline{\frac{\varphi k n}{t^3} \times \frac{V - X}{16} + \frac{\varphi}{t^3} \times \frac{E}{4} - \frac{N}{4}} \times n \times \text{ſin.}\ \frac{4}{n} s$$

$+$, &c.

Deinde, cùm ſit tp, ſive $\dot{x}$ ad rp, ſive $x\dot{s}$, ut velocitas v qua deſcribitur tp ad velocitatem $\frac{1}{x} - U$ qua deſcribitur rp, erit $\dot{x} = v \times \frac{x\dot{s}}{\frac{1}{x} - U}$, ſive, quia valor velocitatis v componitur ex quantitatibus exiguis, $\dot{x} = v\dot{s}$ quamproximè, et $\frac{\dot{x}}{\dot{s}} = v$. Verùm etiam æquatio aſſumpta $x = 1 - Q + K\ \text{coſ.}\ \frac{1}{n} s + L\ \text{coſ.}\ \frac{2}{n} s + M\ \text{coſ.}\ \frac{3}{n} s +$, &c. dat $\frac{\dot{x}}{\dot{s}} = -K \times \frac{1}{n}\ \text{ſin.}\ \frac{1}{n} s - L \times \frac{2}{n}\ \text{ſin.}\ \frac{2}{n} s - M \times \frac{3}{n}\ \text{ſin.}\ \frac{3}{n} s - N \times \frac{4}{n}\ \text{ſin.}\ \frac{4}{n} s$, &c.

Habitis igitur duobus velocitatis v valoribus, eorum termini homologi ſtatuantur æquales, atque inde obtinebuntur quantitates aſſumptæ, nempe

$$K = \frac{\varphi}{t^3} \times \frac{n^2}{n^2 - 1} \times \overline{\overline{2kR - \frac{2t^3}{k^2}} \times \overline{n + \frac{1}{2}} - kT \times \overline{n - \frac{1}{2}} - S}$$

$$L = \frac{\varphi}{2t^3} \times \frac{n^2}{n^2 - 4} \times \overline{kS \times \overline{n + 1} - kV \times \overline{n - 1} - 2T}$$

$$M = \frac{\varphi}{3t^3} \times \frac{n^2}{n^2 - 9} \times \overline{kT \times \overline{n + \frac{3}{2}} - kW \times \overline{n - \frac{1}{2}} - 3V}$$

$$N = \frac{\varphi}{4t^3} \times \frac{n^2}{n^2 - 16} \times \overline{kV \times \overline{n + 2} - kX \times \overline{n - 2} - 4W}$$

&c.

indeque manifeſta fit harum quantitatum progreſſio:

atque hoc pacto habetur ſemper diſtantia x planetæ Q à Sole.

Jam ut definiatur planetæ Q motus verus qui deſignatur per s, dicatur w motus medius, ſive, quod perinde eſt, tempus quo planeta deſcripſerit arcum quemlibet Cr; atque ex demonſtratis eſt $\dot{w} = \frac{x\dot{s}}{\frac{1}{x} - U}$; unde, ſubſtitutis valoribus quantitatum, x, $\frac{1}{x}$, et U, et ſumptâ fluente, emergit

$$w = \overline{1 - 2Q + \frac{\varphi khn}{3}} \times s + 2nK - \frac{\varphi kn^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}}$$

$$\times\ \text{ſin.}\ \frac{1}{n}s + \overline{nL - \frac{\varphi kn^2}{8t^3} \times \overline{S - V}} \times \text{ſin.}\ \frac{2}{n}s$$

$$+ \overline{\frac{2nM}{3} - \frac{\varphi kn^2}{18t^3} \times \overline{T - W}} \times \text{ſin.}\ \frac{3}{n}s$$

$$+ \overline{\frac{nN}{2} - \frac{\varphi kn^2}{32t^3} \times \overline{V - X}} \times \text{ſin.}\ \frac{4}{n}s +, \ \&c. + Z$$

denotante Z quantitatem idoneam ut compleatur fluens. Sed, quia motus verus medio æqualis evadere ſupponitur in qualibet planetarum P et Q conjunctione cum Sole, id eſt, ubi angulus PSQ ſive $\frac{1}{n}s$ æquatur, vel nihilo, vel angulo $r \times 180°$, exhibente r quemvis ex numeris naturalibus 1, 2, 3, 4, &c. erit $Z = \overline{2Q - \frac{\varphi khn}{t^3}} \times s$. Ponantur igitur $F = -2nK + \frac{\varphi kn^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}}$, $G = -nL + \frac{\varphi kn^2}{8t^3} \times \overline{S - V}$, $H = -\frac{2nM}{3} + \frac{\varphi kn^2}{18t^3} \times \overline{T - W}$, $I = -\frac{nN}{2} + \frac{\varphi kn^2}{32t^3}$

×

$+ V - X$, &c. eritque motus verus, ſive $s = w + F$ ſin. $\frac{1}{n}s + G \times$ ſin. $\frac{2}{n}s + H \times$ ſin. $\frac{3}{n}s + I \times$ ſin. $\frac{1}{n}s +$, &c. vel, quia parum admodùm differt motus verus à motu medio $s = w + F \times$ ſin. $\frac{1}{n}w + G \times$ ſin. $\frac{2}{n}w + H \times$ ſin. $\frac{3}{n}w + I \times$ ſin. $\frac{4}{n}w +$, &c. Q. E. I.

COROLL. I.

His ita generatim definitis, ut ſpecialis eliciatur in motu cujuſpiam planetæ inæqualitatum menſura, determinandæ ſunt quantitates aſſumptæ.

Itaque planeta P deſignet Terram, planeta Q Venerem, et quoniam eſt diſtantia Terræ ad diſtantiam Veneris à Sole ut 100000 ad 72333, hæc erit ratio k ad 1, adeoque $k = \frac{100000}{72333}$, $kk + 1 = tt = 2.91129$, $b = \frac{2k}{t^3} = 0.94975$; atque inde per methodum in Prop. Iª. expoſitam prodibunt

$R = 9.3925$ $V = 11.1964$ $Y = 5.3380$
$S = 16.6782$ $W = 8.8504$ $Z = 4.1029$
$T = 13.8877$ $X = 6.9045$ &c.

Tum, exiſtente periodo Terræ annuâ dierum 365.2565, et periodo Veneris dierum 224.701, eſt ex jam dictis $n = \frac{365.2565}{365.2565 - 224.701} = 2.59866$; et cùm gravitas in Solem ſit juxta Newtonum ad gravitatem in Terram, paribus diſtantiis, ut 1 ad $\frac{1}{169282}$, erit $\varphi = \frac{1}{169282}$.

Unde, redactis in numeros formulis in hac propoſitione datis, emergunt

$K = 0.0000103$ $\quad N = -\,0.0000065$
$L = 0.0000444$ $\quad O = -\,0.0000024$
$M = 0.0000377$ $\quad O' = -\,0.0000011$, &c.

Atque ex his tandem deducuntur

$F = -\,0.0000473$ $\quad I = 0.0000100$
$G = -\,0.0001078$ $\quad I' = 0.0000033$
$H = -\,0.0000684$ $\quad$ &c.

Hinc ergo habentur valores coefficientium æquationis $s = w + F \times \text{ſin.}\,\frac{1}{n}w + G \times \text{ſin.}\,\frac{2}{n}w + H \times \text{ſin.}\,\frac{3}{n}w +$, &c. ubi s denotat motum Veneris verum, w motum medium, et $\frac{1}{n}w$ angulum PSQ ſive differentiam longitudinum heliocentricarum Terræ et Veneris; vel, reductis quantitatibus F, G, H, &c. ad exprimendas more aſtronomico circuli partes, fit $s = w - 9''.76 \times \text{ſin.}\,\frac{1}{n}w - 22''.24 \times \text{ſin.}\,\frac{2}{n}w + 14''.11 \times \text{ſin.}\,\frac{3}{n}w + 2''.06 \times \text{ſin.}\,\frac{4}{n}w + 0''.68 \times \text{ſin.}\,\frac{5}{n}w +$, &c.

Ut exemplum apponam, eſto angulus PSQ ſive $\frac{1}{n}w = 40^\circ$, ac prodibit $s = w - 15''.5$; motus igitur medius ſuperat verum, eorumque differentia eſt $15''.5$.

Computatâ hoc pacto differentiâ inter motum Veneris verum et medium reſpectu Solis, ſequenti modo innoteſcet quanta evadat cùm e Terrâ ſpectatur. Eſto

PSQ

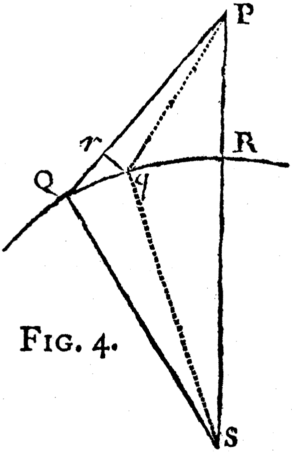

FIG. 4.

PSQ (Fig. 4.) angulus exhibens, ut priùs, differentiam longitudinum planetarum P et Q tempore quovis dato, et in circulo RQ exhibente portionem orbitæ planetæ Q, ſumatur arcus Qq æqualis differentiæ motuum prædictæ, et ductis Sq, Pq, centro P et radio Pq deſcribatur arcus qr ſecans PQ in r, atque, ob parvitatem arcuum Qq, qr, erit $Qq : qr :: \text{rad.} : \text{ſin.}\ PQq$; deinde $\frac{Qq}{QS} : \frac{qr}{PQ} :: \text{ang.}\ QSq : \text{ang.}\ QPq$; adeoque $\frac{\text{rad.}}{QS} : \frac{\text{ſin.}\ PQq}{PQ} :: \text{ang.}\ QSq : \text{ang.}\ QPq$, unde $\text{ang.}\ QPq = \text{ang.}\ QSq \times \frac{QS}{PQ} \times \frac{\text{ſin.}\ PQq}{\text{rad.}}$. Datis igitur angulo PSQ et diſtantiis PS, QS, dabitur diſtantia PQ, et angulus PQS, adeoque et angulus PQq: unde innoteſcet angulus quæſitus QPp, hoc eſt æquatio motûs, prout apparet ſpectatori in centro Terræ locato. Hincque, quamvis ſit modica motûs Veneris inæqualitas telluris actione genita, qualis tamen ſit ut pateat, libet eam in ſequenti tabulâ oculis ſubjicere.

Hujus tabulæ columna prima exhibet angulum QPS, ſive elongationem Veneris à Sole mediam; ſecunda indicat correctionem hujus elongationis, à conjunctione Veneris inferiore uſque ad maximam ejus elongationem quæ in orbe circulari eſt 46° 19′ 50″ circiter. Tertia et quarta columna eodem modo exhibent elongationem Veneris, ejuſque correctionem, à tempore elongationis maximæ uſque ad conjunctionem ſuperiorem.

Elongatio Ven. à Sole.	Correctio.	Elongatio Ven. à Sole.	Correctio.
° ′ ″	″	° ′ ″	″
0	0	46 19 50	0
5	0	46	+ 2.3
10	0	45	5.1
15	0	40	9.5
20	— 0.5	35	7.3
25	0.8	30	1.8
30	1.5	25	— 4.4
35	2.8	20	9.2
40	2.9	15	11.2
45	2.7	10	10.2
46	1.7	5	6.0
46 19 50	0	0	0

Exempli gratiâ, ſi Venus à conjunctione inferiore digreſſa motu ſuo medio diſceſſerit à Sole angulo elongationis 40°, erit vera Veneris elongatio $40^\circ - 2''.9 = 39^\circ\ 59'\ 57''.1$: pariter, ſi ulteriùs delata Venus pervenerit ad eamdem elongationem 40°, erit tunc vera Veneris elongatio $40^\circ\ 0'\ 9''.5$. Eædem omninò ſunt correctiones et cum iiſdem ſignis adhibendæ, ubi poſt conjunctionem ſuperiorem eædem eveniunt elongationes.

COROLL. II.

Ex præcedentibus etiàm deducitur diſtantia Veneris à Sole pro quolibet ejus cum Terrâ et Sole aſpectu, in hypotheſi quod, ſecluſâ Terræ attractione, in orbitâ circulari revolveret. Sic, ſi angulus $\frac{1}{n}s$, ſeu PSQ ſit 90°, vel 270°, æquatio $x = 1 - Q + K \cos. \frac{1}{n}s + L \cos. \frac{2}{n}s + M \cos. \frac{3}{n}s + N \cos. \frac{4}{n}s +$, &c. fit $x = 0.9999437$ circiter; et ſi ſit PSQ $= 180°$, fit $x = 1.0000607$.

Unde, ſi diſtantia Veneris à Sole in conjunctione inferiore ponatur - - -	10000000
In quadraturis cum Terrâ erit ipſius diſtantia - - - - - - - - - -	9999437
In conjunctione ſuperiore erit - - -	10000607

Item innoteſcit differentia inter tempus periodicum Veneris, quale nunc eſt, et tempus illud periodicum, quale foret, ſi unicâ Solis attractione in orbe circulari moveretur. Siquidem, cùm Venus poſt diſceſſum ſuum à conjunctione ad eamdem redierit, æquatio generalis in propoſitione tradita, quæ exprimit relationem inter motum Veneris verum et medium, evadit

$w = \overline{1 - 2Q + \frac{\varphi k h n}{t^3}} \times s$, ſive $w = 1.0000066 \times s$

circiter: unde tempus periodicum Veneris eſt ad tempus illud alterum periodicum, ut 1.0000066 ad 1; adeóque, ſi nulla foret gravitatio Veneris in Terram, revolutionem ſuam circa Solem minutis duobus horæ primis citiùs perageret.

PROPOSITIO III. PROBLEMA.

In ſyſtemate duorum planetarum in orbitis circularibus circa Solem revolventium, motum nodorum orbitæ planetæ interioris, quatenus ex vi planetæ exterioris oritur, inveſtigare.

Per motum nodorum hîc intelligendus eſt motus interſectionis plani orbis planetæ interioris cum plano orbis planetæ exterioris ſpectato ut immoto. Itaque eſto Sol in S (Fig. 5.) et centro S atque radio SQ deſcribantur in ſuperficie ſphæræ duo circuli QN, PN, ſeſe interſecantes in N, quorum prior QN deſignet ſitum plani orbis planetæ interioris Q, et poſterior PN ſitum plani orbis planetæ exterioris, cujus locus ſit in rectâ SP productâ. Eodem centro S et radio SP deſcribatur circulus PK, cujus planum ſit plano

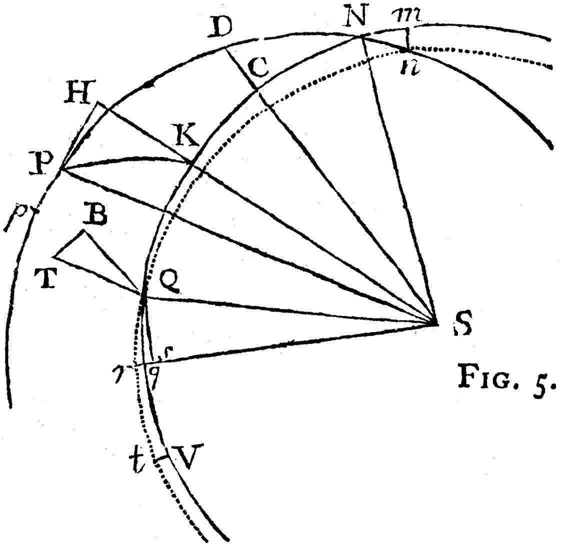

FIG. 5.

SQN perpendiculare, ſecetque circulum QN in K, et in SK demittatur perpendiculum PH: tum ductâ QT parallelâ rectæ SP et TB in planum SQN normali, ſi linea QT exhibeat vim qua trahitur planeta Q in directione QT, ſeu SP, TB exhibebit vim qua diſtrahitur perpendiculariter à plano ſuæ orbitæ; eritque triangulum QTB ſimile triangulo SPH, atque adeò, $TB : QT :: PH : SP :: \text{ſin.}\ PK : 1$; deinde in triangulo ſphærico rectangulo PKN habetur, $1 : \text{ſin.}\ PN :: \text{ſin.}\ PNK : \text{ſin.}\ PK$; unde, conjunctis rationibus, et ſcripto c pro ſinu anguli PNK ad radium 1, hoc eſt, pro ſinu inclinationis orbis QN ad orbem PN, provenit $TB = QT \times c \times \text{ſin.}\ PN$. Sumatur jam arcus quàm minimus Qq, ad quem erigitur lineola perpendicularis qr, æqualis duplo ſpatio quod planeta Q percurrere poſſet impellente vi TB quo tempore in orbe ſuo deſcriberet arcum illum Qq, et centro S deſcriptus circulus rQn ſecans circulum PN in n exhibebit ſitum orbis planetæ Q poſt tempus illud, nodo N tranſlato in n; atque in QN demiſſo perpendiculo nm, et in Sq perpendiculo Qs, erit angulus qQr, ſive NQn ad duplum angulum qQs, id eſt, ad angulum QSq, ut vis TB ad gravitatem (nempe 1) planetæ Q in Solem; hoc eſt, $\frac{nm}{\text{ſin.}\ QN} : Qq :: TB : 1$; in triangulo autem rectangulo Nmn, eſt $Nn : nm :: 1 : c$; quarè conjunctis his rationibus, prodit $Nn = \frac{TB \times \text{ſin.}\ QN \times Qq}{c}$; ſed ſuprà invenimus $TB = QT \times c \times \text{ſin.}\ PN$, unde fit $Nn = QT \times \text{ſin.}\ PN \times \text{ſin.}\ QN \times Qq$.

Eſto SC linea conjunctionis planetarum, fiatque, ut in propoſitione præcedente, arcus $CQ = s$, $Qq = \dot{s}$, SQ $= 1$; et, quia inclinatio orbis QN ad orbem PN exigua ſupponitur, erit etiam hîc ang. PSQ $= \frac{1}{n}s$ quamproximè; proindeque, poſito arcu CN $= a$, erit QN $= s + a$ et PN $= s - \frac{1}{n}s + a$ quamproximè. Porrò, cùm lentiſſimè moveantur nodi, arcus CN ſpectari poteſt quaſi invariabilis per multarum planetæ Q revolutionum ſeriem, atque adeò fluxio arcus QN eadem erit cum fluxione arcûs QC. His poſitis, habebitur ſin. PN $\times$ ſin. QN $= \frac{1}{2}$ coſ. $\frac{1}{n}s - \frac{1}{2}$ coſ. $\overline{2s - \frac{1}{n}s + 2a}$, eſtque per propoſitionem primam QT $= \frac{\varphi k}{z^3} - \frac{\varphi}{k^2} = \frac{\varphi k}{t^3}$ in $R - \frac{t^3}{k^3} + S$ coſ. $\frac{1}{n}s + T$ coſ. $\frac{2}{n}s + V$ coſ. $\frac{3}{n}s + W$ coſ. $\frac{4}{n}s +$, &c. unde ſubſtitutis his valoribus in æquatione Nn $=$ QT $\times$ ſin. PN $\times$ ſin. QN $\times$ Qq, et ſumptâ fluente per methodum in Coroll. IV. lemmatis edoctam, prodibit ſumma omnium Nn, ſive motus nodi, quo tempore planeta Q à loco conjunctionis C procedens in orbe ſuo deſcripſerit arcum CQ, æqualis $\frac{\varphi k n}{2t^3}$ in $\frac{S}{2n}s + \overline{R - \frac{t^3}{k^3} + \frac{T}{2}} \times$ ſin. $\frac{1}{n}s + \frac{S+V}{4}$ ſin. $\frac{2}{n}s + \frac{T+W}{6}$ ſin. $\frac{3}{n}s +$, &c. $+ \frac{\varphi k n}{2t^3}$ in Z $\times$ ſin. $2a - \overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n-1}$ ſin. $\overline{2s - \frac{1}{n}s + 2a} - \frac{S}{2} \times \frac{1}{2n}$ ſin. $\overline{2s + 2a} - \frac{S}{2} \times \frac{1}{2n-2}$ ſin. $\overline{2s - \frac{2}{n}s + 2a} - \frac{T}{2}$

$\times$

$$\times\frac{1}{2n-3}\,\text{ſin.}\;\overline{2s-\frac{3}{n}s+2a}-\frac{T}{2}\times\frac{1}{2n+1}\,\text{ſin.}\;\overline{2s+\frac{1}{n}s+2a}-\frac{V}{2}\times\frac{1}{2n-4}\,\text{ſin.}\;\overline{2s-\frac{4}{n}s+2a}$$

$$-\frac{V}{2}\times\frac{1}{2n+2}\,\text{ſin.}\;\overline{2s+\frac{2}{n}s+2a}$$, &c. exiſtente

$$Z=\overline{2n-1}\text{ in }R-\frac{t^3}{k^3}\times\frac{1}{\overline{2n-1}|^2}+\frac{S}{\overline{2n-2}+2n}+\frac{T}{\overline{2n-3}\times\overline{2n+1}}+\frac{V}{\overline{2n-4}\times\overline{2n+2}}+\frac{W}{\overline{2n-5}\times\overline{2n+3}}$$

+, &c. atque in his ſeriebus patet terminorum progreſſio. Q. E. I.

COROLL. I.

Hic liquet multas oriri in motu nodorum æquationes; ſed quia minutæ ſunt, et locum planetæ Q ferè nihil mutant, ideò ſatis erit rationem habere motûs nodorum medii et æquationis ſolîus *periodicæ*, qui ſic ex præcedentibus deducuntur. Cùm in planis parùm ad ſe inclinatis moveri ſupponantur planetæ P et Q, quoties revertentur ad conjunctionem, angulus PSQ, ſive $\frac{1}{n}s$, qui metitur eorum diſtantiam à ſe invicem, evadet = 360° vel = $r\times 360°$, exiſtente r numero integro: et quia, ſumpto arcu quolibet A, eſt ſemper ſin. $\overline{r\times 360° + A}$ = ſin. A; hinc, ſi computatur motus nodi pro tempore conjunctionum, expreſſio illa generalis et prolixa in propoſitione tradita in hanc ſimplicem abit $\frac{\varphi k}{2t^3}\times\overline{\frac{S}{2}s-nZ\times\overline{\text{ſin.}\,\overline{2s+2a}-\text{ſin.}\,2a}}$, ſive per Coroll. I. lemmatis

$$\frac{\varphi k}{2t^3}\times\overline{\frac{S}{2}s-2nZ\times\text{ſin.}\,s\times\text{coſ.}\,\overline{s+2a}}.$$

Hic eſt igitur motus nodorum factus, quo tempore planetæ P et Q à conjunctione provecti poſt quotlibetcunque revolutiones ad conjunctionem quamvis aliam pervenerint, exhibente s arcum à planetâ Q in ſuâ orbitâ intereà deſcriptum. Terminus $\frac{\phi k}{2t^3} \times 2nZ \times$ ſin. $s \times$ coſ. $\overline{s + 2a}$ indicat æquationem *periodicam* et facillimè computatur: cùmque hæc æquatio modò ſit additiva, modò ſubtractiva, patet termino altero $\frac{\phi k}{2t^3} \times \frac{S}{2} s$ exprimi generatìm motum nodi medium.

C O R O L L. II.

Eſto planeta P Terra, Q Venus, et revolutionem Veneris ab unâ conjunctione inferiore cum Terrâ ad alteram vocemus, brevitatis gratiâ, revolutionem *ſynodicam*; eritque poſt unam revolutionem ſynodicam $\frac{1}{n} s = 360°$, proindeque $s = n \times 360° = 935° \, 31'$; hic igitur eſt arcus deſcriptus à Venere inter duas ejuſdem generis conjunctiones. Hinc motus nodi medius tempore revolutionis unius ſynodicæ, qui juxta corollarium præcedens eſt $\frac{\phi k S}{4t^3} s$ fit $\frac{\phi k n S}{4t^3} = 360° = 23''.087$; atque hic motus imminutus in ratione temporis periodici Terræ circa Solem ad revolutionem Veneris ſynodicam, id eſt, in ratione 1 ad $n - 1$, evadit $14''.44$, motus ſcilicet annuus nodorum Veneris regreſſivus, qui ſpatio centum annorum fit $24' \, 4''$.

Æquatio periodica $\frac{\phi k n Z}{t^3} \times$ ſin. $s \times$ coſ. $\overline{s + 2a}$ ut adhuc ſimplicior evadat, ponamus arcum a ſive CN perexiguum eſſe vel nullum, id eſt, ſupponamus conjunctionem Terræ et Veneris fieri proximè in nodo, quemadmodum contingit hoc anno 1761, eritque æquatio periodica $\frac{\varphi k n Z}{t^3} \times$ ſin. $s \times$ coſ. $s = \frac{\varphi k n Z}{2 t^3} \times$ ſin. $2s$. Cùm igitur ſit $Z = 32.33$ circiter, formula $\frac{\varphi k S}{4 t^3} s - \frac{\varphi k n Z}{2 t^3}$ ſin. $2s$, quæ per corollarium præcedens exprimit generatìm motum nodi in qualibet ſerie revolutionum ſynodicarum confectum, fit $0.000006855 \times s - 14''.2 \times$ ſin. $2s$. Æquatio igitur periodica $14'.2 \times$ ſin. $2s$, quam *generalem* voco, eſt ut ſinus dupli arcûs à Venere deſcripti in datâ ſerie revolutionum ſynodicarum, nec ultra $14''.2$ aſcendit. Jam, ſi pro s ſubſtituatur $935° \; 31'$, erit ſin. $2s =$ ſin. $71° \; 2'$, et regredientur nodi, in primâ revolutione ſynodicâ poſt conjunctionem factam in nodo, per arcum $23'' - 14''.2 \times$ ſin. $71° \; 2' = 10''$: et, ſi r denotet numerum quemcumque revolutionum ſynodicarum, motus nodi, peractis illis revolutionibus, erit $r \times 23' - 14''.2 \times$ ſin. $r \times 71° \; 2'$; pariterque, peractis revolutionibus quarum numerus eſt $r - 1$, idem motus erit $\overline{r - 1} \times 23'' - 14''.2 \times$ ſin. $\overline{r - 1} \times 71° \; 2'$; poſterior motus ex priore auferatur, et remanebit $23'' - 14''.2 \times \overline{\text{ſin.}\, r \times 71° \, 2' - \text{ſin.}\, \overline{r - 1} \times 71° \, 2'}$ $= 23'' - 14''.2 \times 2$ ſin. $35° \; 31' \times$ coſ. $\overline{r \times 71° \, 2' - 35° \, 31'}$ $= 23'' - 16''.5 \times$ coſ. $\overline{2r - 1} \times 35° \; 31'$ pro motu nodi facto, tempore illius revolutionis ſynodicæ, cujus locum in ſerie revolutionum indicat numerus r. Exempli gratiâ, ſi deſideretur motus nodi tempore revolutionis quartæ ſynodicæ poſt conjunctionem factam in nodo, erit $r = 4$, et regreſſus nodi erit

$23'' - 16''.5 \times \text{coſ.}\ 7 \times 35^\circ\ 31 = 29''$. Sic ope hujus formulæ $23'' - 16''.5 \times \text{coſ.}\ \overline{2r - 1} \times 35^\circ\ 31'$ facilè computatur ſequens tabula, quæ exhibet regreſſum nodi Veneris in plano eclipticæ, pro duodecim ſigillatim revolutionibus ſynodicis quæ proximè ſequuntur conjunctionem Terræ et Veneris factam in nodo vel proximè ad nodum.

In revol. Ven. ſynod.	Regreſſus nodi Ven.	In revol. Ven. ſynod.	Regreſſus nodi Ven.
	$''$		$''$
1^{a}.	10	7^{a}.	26
2^{a}.	28	8^{a}.	39
3^{a}.	39	9^{a}.	30
4^{a}.	29	10^{a}.	11
5^{a}.	10	11^{a}.	8
6^{a}.	9	12^{a}.	25

Qui motus poteſt, cùm libuerit, ad annos communes reduci.

Denique patet æquationem periodicam, nempe $16''.5 \times \text{coſ.}\ \overline{2r - 1} \times 35^\circ\ 31'$, quam *ſpecialem* appello, ubi maxima eſt, evadere $16''\frac{1}{2}$; ac proinde regreſſum nodi in unâ revolutione ſynodicâ nuſquam ſuperare $39''\frac{1}{2}$, nec minorem eſſe $6''\frac{1}{2}$.

PROPOSITIO IV. PROBLEMA.

Iiſdem poſitis, variationem inclinationis orbis planetæ interioris ad planum orbis planetæ exterioris determinare.

Eſto NQV (Fig. 5.) quadrans circuli, cui erigatur perpendicularis Vt occurrens arcui nQr producto in t, eritque Vt menſura variationis inclinationis orbis NQV factæ quo tempore nodus N transfertur in n. Eſt autem Vt : nm :: ſin. QV ſive coſ. QN : ſin. QN, atque nm : Nn :: c : 1, c denotante ſinum inclinationis orbis QN ad orbem PN, adeoque Vt : Nn :: c $\times$ coſ. QN : ſin. QN; unde $\mathrm{V}t = \mathrm{N}n \times \frac{c \times \text{coſ. QN}}{\text{ſin. QN}}$, ſive, quia per propoſitionem ſuperiorem habetur $\mathrm{N}n = \mathrm{QT} \times \text{ſin. PN} \times \text{ſin. QN} \times \mathrm{Q}q$, $\mathrm{V}t = c \times \mathrm{QT} \times \text{ſin. PN} \times \text{coſ. QN} \times \mathrm{Q}q$. Hinc, cum ſit $\text{ſin. PN} \times \text{coſ. QN} = \frac{1}{2}\,\text{ſin.}\,\overline{2s - \frac{1}{n}s + 2a} - \frac{1}{2}\,\text{ſin.}\,\frac{1}{n}s$, ſumptâ fluente prodit variatio inclinationis, quo tempore planeta Q à loco conjunctionis C movetur per arcum CQ, æqualis $-\frac{\varphi c k n}{2t^3}$ in $\overline{\mathrm{R} - \frac{t^3}{k^3} - \frac{\mathrm{T}}{2}} \times \text{ſin. verſ.}\,\frac{1}{n}s + \frac{\mathrm{S}-\mathrm{V}}{4}\,\text{ſin. verſ.}\,\frac{2}{n}s + \frac{\mathrm{T}-\mathrm{W}}{6}\,\text{ſin. verſ.}\,\frac{3}{n}s + \frac{\mathrm{V}-\mathrm{X}}{8}\,\text{ſin. verſ.}\,\frac{4}{n}s +$, &c. $+ \frac{\varphi c k n}{2t^3}$ in $-\mathrm{Z} \times \text{ſin. verſ.}\,2a + \overline{\mathrm{R} - \frac{t^3}{k^3}} \times \frac{1}{2n-1}\,\text{ſin. verſ.}\,\overline{2s - \frac{1}{n}s + 2a} + \frac{\mathrm{S}}{2} \times \frac{1}{2n}\,\text{ſin. verſ.}\,\overline{2s + 2a} + \frac{\mathrm{S}}{2} \times \frac{1}{2n-2}\,\text{ſin. verſ.}$

2 s

$$\overline{2s - \tfrac{2}{n}s + 2a} + \tfrac{\mathrm{T}}{2} \times \tfrac{1}{2n+1}\ \text{ſin. verſ.}\ \overline{2s + \tfrac{1}{n}s + 2a}$$

$$+ \tfrac{\mathrm{T}}{2} \times \tfrac{1}{2n-3}\ \text{ſin. verſ.}\ \overline{2s - \tfrac{3}{n}s + 2a} + \tfrac{\mathrm{V}}{2} \times \tfrac{1}{2n+2}$$

$$\text{ſin. verſ.}\ \overline{2s + \tfrac{2}{n}s + 2a} + \tfrac{\mathrm{V}}{2} \times \tfrac{1}{2n-4}\ \text{ſin. verſ.}$$

$$\overline{2s - \tfrac{4}{n}s + 2a} + \tfrac{\mathrm{W}}{2} \times \tfrac{1}{2n+3}\ \text{ſin. verſ.}\ \overline{2s + \tfrac{3}{n}s + 2a}$$

$$+ \tfrac{\mathrm{W}}{2} \times \tfrac{1}{2n-5}\ \text{ſin. verſ.}\ \overline{2s - \tfrac{5}{n}s + 2a},\ \text{\&c.}$$

Exiſtente hîc eodem valore quantitatis Z ac in propoſitione præcedente. Q. E. I.

COROLL.

Si computetur variatio inclinationis pro tempore conjunctionum, facilè obtinebitur; hæc enim per formulam in propoſitione traditam evadit $\frac{\varphi c k n}{2t^3} \times \mathrm{Z}$ $\times\ \overline{\text{ſin. verſ.}\ \overline{2s + 2a} - \text{ſin. verſ.}\ 2a}$ quæ itèm, ſi prima conjunctionum, à qua ſumitur motûs exordium, ſtatuatur in nodo, fit $\frac{\varphi c k n}{1t^3} \times \mathrm{Z} \times$ ſin. verſ. $2s$.

Hoc eſt igitur decrementum inclinationis orbis planetæ Q factum in qualibet ſerie revolutionum ad conjunctionem, deſignante s arcum intereà à planetâ circa Solem deſcriptum. Conferatur hæc inclinationis variatio cum æquatione nodi periodicâ eodem tempore genitâ, prout in propoſitione ſuperiore definitur, et patebit priorem eſſe ad poſteriorem ut $c \times$ ſin. verſ. $2s$ ad ſin. $2s$.

Ut ad orbem Veneris hæc transferantur, quem ſi inclinari ad orbem Terræ ſupponatur angulo $3° 23' 20''$, erit $\frac{\varphi c k n}{2 t^3} \times Z \times$ ſin. verſ. $2s = 0''.84 \times$ ſin. verſ. $2s$. Unde palàm fit: 1°. in quacumque ſerie revolutionum ſynodicarum, poſt conjunctionem factam in nodo, decrementum inclinationis orbitæ Veneris ad eclipticam non ſuperare $2 \times 0''.84 = 1''.68$, quod è Terrâ ſpectatum evadit $4''.4$: 2°. cùm, peractâ unâ revolutione ſynodicâ, ſit ſin. verſ. $2s =$ ſin verſ. $71^\circ 2'$, inclinationis decrementum pro qualibet ſerie revolutionum ſynodicarum quarum numerus eſt r, eſſe $0''.84 \times$ ſin. verſ. $r \times 71^\circ\ 2'$, et pro ſerie revolutionum quarum numerus eſt $r - 1$, eſſe $0''.84 \times$ ſin. verſ. $\overline{r - 1} \times 71^\circ\ 2'$; unde horum decrementorum differentia $\overline{0''.84 \times \text{ſin. verſ. } r \times 71^\circ\ 2' - \text{ſin. verſ. } \overline{r - 1} \times 71^\circ\ 2} = 0''.84 \times \overline{2 \text{ ſin. } 35^\circ}\ 31' \times \text{ſin. } \overline{2r - 1} \times 35^\circ\ 31' = 0''.98 \times \text{ſin. } \overline{2r - 1} \times 35^\circ\ 31'$, exprimit variationem inclinationis genitam tempore revolutionis ſynodicæ illius, cujus locum in ſerie revolutionum denotat numerus r: atque hæc variatio, ut patet, nuſquam excedit $0''.98$ è Sole conſpecta, quæ ſpectatori in centro Terræ collocato ſub angulo $2''\frac{1}{2}$ apparebit. Cum igitur tantilla ſit orbitæ Veneris inclinationis variatio, non videtur operæ pretium de eâ ulteriùs exquirere.

Demonſtratis, quæ ad perturbationem motûs planetæ interioris ſpectant, ſupereſt ut, quibus perturbationibus afficiatur motus planetæ exterioris, viciſſim expendamus.

PROPOSITIO V. PROBLEMA.

In ſyſtemate duorum planetarum circa Solem in orbibus penè circularibus revolventium, determinare vim planetæ interioris ad perturbandum motum exterioris.

Simili ratiocinio ei, quod in propoſitione primâ uſurpavimus, etiam hoc problema ſolvitur. Itaque poſitâ unitate pro diſtantiâ planetæ P à Sole, ubi ambo planetæ P et Q conjunguntur cum Sole, (Fig. 1.) fiat $SP = x$, $SQ = k$, $PQ = z$. Sit 1 ad φ ut gravitatio planetæ P in Solem in diſtantiâ 1 ad ejuſdem planetæ P gravitationem in planetam Q in eâdem diſtantiâ, eritque $\frac{\varphi}{z^2}$ gravitas planetæ P in planetam Q in diſtantiâ PQ. Productâ, ſi opus eſt, PQ ad O ut ſit $PO = \frac{\varphi}{z^2}$, et ductâ OI parallelâ rectæ QS occurrente PS productæ in I, reſolvatur vis PO in vires PI et OI, eritque propter ſimilia triangula PQS, POI, vis $OI = \frac{PO \times QS}{PQ} = \frac{\varphi k}{z^3}$, atque vis $PI = \frac{PO \times PS}{PQ} = \frac{\varphi x}{z^3}$ ſive vis $PI = \frac{\varphi}{z^3}$ quamproximè. Vis OI impellit planetam P in directione parallelâ rectæ SQ, et in eundem ſenſum urgetur Sol vi $\frac{\varphi}{k^2}$ qua gravitat in planetam Q: exceſſu igitur ſolo vis prioris ſupra poſteriorem, nempe $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$, cenſendus eſt urgeri planeta P in directione parallelâ rectæ SQ.

Porrò vis $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$ ea pars, quæ agit perpendiculariter ad radium PS, eſt $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. PSQ, atque altera pars, quæ amovet planetam P à Sole ſecundum PS, eſt $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ coſ. PSQ. Auferatur hæc poſterior vis ex vi PI, et manebit vis $\frac{\varphi}{z^3} + \overline{\frac{\varphi}{k^2} - \frac{\varphi k}{z^3}} \times$ coſ. PSQ, qua planeta P urgetur verſus Solem.

Eſto DCS (Fig. 2.) linea conjunctionis planetarum, et arcus DP, ſive angulus DSP vocetur s, denotentque P et Q reſpectivè tempora periodica planetarum P et Q, eritque, poſito $n = \frac{Q}{P - Q}$, ang. PSQ $= \frac{1}{n}s$. Tum, ſi fiat $t^2 = 1 + kk$, et $b = \frac{2k}{t^2}$, erit uti in Prop. I. expoſuimus, $z^2 = t^2 \times \overline{1 - b \text{ coſ. } \frac{1}{n}s}$, atque $\frac{1}{z^3} = \frac{1}{t^3} \times \overline{R + S \text{ coſ. } \frac{1}{n}s + T \text{ coſ. } \frac{2}{n}s + V \times \text{coſ. } \frac{3}{n}s +, \text{\&c.}}$ et quemadmodùm ibi erat $b = \frac{2PS \times SQ}{\overline{PS}^2 + \overline{SQ}^2}$, hîc item eſt $b = \frac{2PS \times SQ}{\overline{PS}^2 + \overline{SQ}^2}$, adeoque valores quantitatum aſſumptarum R, S, T, &c. iidem hîc ſunt ac in propoſitione primâ.

Unde vis $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. PSQ, qua ſollicitatur planeta P in directione ad radium PS perpendiculari, ſic exprimetur $\frac{\varphi k}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times$ ſin. $\frac{1}{n}s + \frac{S - V}{2}$

ſin. $\frac{2}{n}s + \frac{T - W}{2}$ ſin. $\frac{3}{n}s + \frac{V - X}{2}$ ſin. $\frac{4}{n}s +$, &c. Et vis $\frac{\varphi}{z^3} + \overline{\frac{\varphi}{k^2} - \frac{\varphi k}{z^3}} \times$ coſ. PSQ, qua urgetur planeta P in Solem ſecundum radium PS, fiet $\frac{\varphi}{t^3}$ in $R - \frac{kS}{2} - \overline{kR + \frac{kT}{2} - \frac{t^3}{k^2} - S} \times$ coſ. $\frac{1}{n}s - \frac{kS + kV - 2T}{2}$ coſ. $\frac{2}{n}s - \frac{kT + kW - 2V}{2}$ coſ. $\frac{3}{n}s - \frac{kV + kX - 2W}{2}$ coſ. $\frac{4}{n}s +$, &c. Q. E. I.

PROPOSITIO VI. PROBLEMA.

Inæqualitates motûs planetæ exterioris ex viribus prædictis ortas inveſtigare.

Per analyſim in propoſitione ſecundâ inſtitutam vis ad radium PS perpendicularis generabit accelerationem, vel retardationem velocitatis, dum arcus quilibet DP deſcribitur à planeta P, æqualem $\frac{\varphi k n}{t^3}$ in $b - \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times$ coſ. $\frac{1}{n}s - \frac{S - V}{4}$ coſ. $\frac{2}{n}s - \frac{T - W}{6}$ coſ. $\frac{3}{n}s - \frac{V - X}{8}$ coſ. $\frac{4}{n}s -$, &c. $= U$ exiſtente $b = R - \frac{t^3}{k^3} - \frac{T}{2} + \frac{S - V}{4} + \frac{T - W}{6} + \frac{V - X}{8} +$, &c.

Deinde ſi ſcribatur p pro vi illâ planetæ Q qua urgetur planeta P in Solem, prout in propoſitione præcedente definita eſt, et v pro velocitate aſcenſûs vel deſcenſûs planetæ P ſecundum radium PS, et jam ſupponatur $SP = x = 1 - Q + K \text{ coſ. } \frac{1}{n}s + L \text{ coſ. } \frac{2}{n}s + M \text{ coſ. } \frac{3}{n}s + N \text{ coſ. } \frac{4}{n}s +$, &c. exiſtente $Q = K + L + M + N +$, &c. erit $\frac{1}{x^2} + p$ vis centripeta planetæ P, et $\frac{1}{x} \times \overline{\frac{1}{x} + U}|^2$ ejuſdem vis centrifuga, atque inde habebitur $\dot{v} = \overline{\frac{1}{x^2} + p - \frac{1}{x} \times \overline{\frac{1}{x} - U}|^2} \times \frac{x\dot{s}}{\frac{1}{x} + U}$.

Tum reſtitutis valoribus quantitatum U, p, x, et proſequendo calculum prout in Prop. II. poſitis

$$A = Kn + \frac{2\varphi kn^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} - \frac{\varphi n}{t^3} \times \overline{kR - \frac{t^3}{k^2} - S + \frac{kT}{2}}$$

$$B = L \times \frac{n}{2} + \frac{\varphi kn^2}{4t^3} \times \overline{S - V} - \frac{\varphi n}{4t^3} \times \overline{kS + kV - 2T}$$

$$C = M \times \frac{n}{3} + \frac{\varphi kn^2}{9t^3} \times \overline{T - W} - \frac{\varphi n}{6t^3} \times \overline{kT + kW - 2V}$$

$$D = N \times \frac{n}{4} + \frac{\varphi kn^2}{16t^3} \times \overline{V - X} - \frac{\varphi n}{8t^3} \times \overline{kV + kX - 2W}$$

&c.

prodibit $v = \overline{\frac{\varphi}{t^3} \times \overline{R - \frac{kS}{2}} - \frac{2\varphi khn}{t^3} - Q} \times s + A \times \text{ſin. } \frac{1}{n}s + B \times \text{ſin. } \frac{2}{n}s + C \times \text{ſin. } \frac{3}{n}s + D \times \text{ſin. } \frac{4}{n}s +$, &c. $+ Z$, et factâ hypotheſi quòd ſit $v = 0$ ubi angulus $PSQ = 0$, vel $r \times 180°$, exprimente r unum ex numeris naturalibus 1, 2, 3, 4, &c. erit $Z = -\overline{\frac{\varphi}{t^3} \times \overline{R - \frac{kS}{2}} - \frac{2\varphi khn}{t^3} - Q} \times s$, ac proinde $v = A \times \text{ſin.}\,\frac{1}{n}s + B \times \text{ſin.}\,\frac{2}{n}s + C \times \text{ſin.}\,\frac{3}{n}s + D \times \frac{4}{n}s +$, &c.

Tùm, quia vis centripeta hîc excedere ſupponitur vim centrifugam, cùm contrarium ſuppoſitum fuerit in propoſitione ſecundâ, habetur $-\dot{x} = v \times \frac{xs}{\frac{1}{x} + U}$ ſive $-\dot{x} = v\dot{s}$ proximè, et $-\frac{\dot{x}}{\dot{s}} = v = K \times \frac{1}{n}\,\text{ſin.}\,\frac{1}{n}s + L \times \frac{2}{n}\,\text{ſin.}\,\frac{2}{n}s + M \times \frac{3}{n}\,\text{ſin.}\,\frac{3}{n}s + N \times \frac{4}{n}\,\text{ſin.}\,\frac{4}{n}s +$, &c.

Unde factâ collatione terminorum hujus valoris velocitatis v cum terminis homologis valoris ſupra inventi, emergent

$$K = -\frac{\varphi}{t^3} \times \frac{n^2}{n^2-1} \times \overline{\overline{2kR - \frac{2t^3}{k^2}} \times \overline{n-\tfrac{1}{2}} - kT \times \overline{n+\tfrac{1}{2}} + S}$$

$$L = -\frac{\varphi}{2t^3} \times \frac{n^2}{n^2-4} \times \overline{kS \times \overline{n-1} - kV \times \overline{n+1} + 2T}$$

$$M = -\frac{\varphi}{3t^3} \times \frac{n^2}{n^2-9} \times \overline{kT \times \overline{n-\tfrac{3}{2}} - kW \times \overline{n+\tfrac{3}{2}} + 3V}$$

$$N = -\frac{\varphi}{4t^3} \times \frac{n^2}{n^2-16} \times \overline{kV \times \overline{n-2} - kX \times \overline{n+2} + 4W}$$

&c.

atque ità patet hujuſmodi quantitatum progreſſio. Innoteſcet igitur x, ſeu diſtantia planetæ P à Sole in quovis ejus cum planetâ Q aſpectu.

Ut obtineatur planetæ P motus verus s, deſignet w motum medium, et cùm ſit $\dot{w} = \frac{xs}{\frac{1}{x} + U}$, ſubſti-

tuantur

quantur valores quantitatum x, U, et ſumptâ fluente, poſitis

$$F = 2nK + \frac{\varphi k n^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}}$$

$$G = nL + \frac{\varphi k n^2}{8t^3} \times \overline{S - V}$$

$$H = \frac{2nM}{3} + \frac{\varphi k n^2}{18t^3} \times \overline{T - W}$$

$$I = \frac{nN}{2} + \frac{\varphi k n^2}{32t^3} \times \overline{V - X}$$

&c.

proveniet $w = \overline{1 - 2Q - \frac{\varphi k h n}{t^3}} \times s + F \times \text{ſin.}\ \frac{1}{n}s + G \times \text{ſin.}\ \frac{2}{n}s + H \times \text{ſin.}\ \frac{3}{n}s + I \times \text{ſin.}\ \frac{4}{n}s +$, &c. $+ Z$.

Et factâ hypotheſi quod motus verus coincidat cum medio ubi eſt $\frac{1}{n}s$, ſeu angulus PSQ $= 0$, vel $= r \times 180°$, exhibente r quemvis ex numeris 1, 2, 3, 4, &c. erit $Z = \overline{2Q + \frac{\varphi k h n}{t^3}} \times s$; ac proinde, ſcriptis $\frac{1}{n}w$, $\frac{2}{n}w$, &c. pro $\frac{1}{n}s$, $\frac{2}{n}s$, &c. quia parùm admodùm differt motus verus à medio, habetur motus verus, ſive $s = w - F \times \text{ſin.}\ \frac{1}{n}w - G \times \text{ſin.}\ \frac{2}{n}w - H \times \text{ſin.}\ \frac{3}{n}w - I \times \text{ſin.}\ \frac{4}{n}w -$, &c. Q. E. I.

COROLL. I.

Deſignet jam planeta P Terram, Q Venerem, et quia poſuimus eſſe diſtantiam mediocrem Terræ à

Sole ad diſtantiam mediocrem Veneris à Sole ut 1 ad k, erit hîc $k = 0.72333$, atque $t = \sqrt{1 + kk} = 1.234182$. Item eſt $n = \frac{Q}{P - Q} = \frac{224.701}{365.2565 - 224.701} = 1.59866$. Quantitates b, R, S, T, &c. eoſdem hîc retinent valores quos habebant in Coroll. I. Prop. II. Verùm, ut motuum Terreſtrium accurata inſtitueretur computatio, dignoſcere neceſſe eſſet effectus aliquos ab actione Veneris provenientes, ex quibus derivare liceret vim attractivam iſtius planetæ, ſed quia ſpeciales hujuſmodi effectus nulli, quantùm noverimus, obſervationibus aſtronomicis explorati habentur, propterea vim Veneris nunc conjecturâ definiemus, ut inde inæqualitates in motu Telluris computatæ, atque cum obſervationibus aſtronomicis collatæ inſervire poſthac poſſint ad eamdem vim certiùs determinandam. Itaque ſupponemus gravitatem in Solem eſſe ad gravitatem in Venerem, paribus diſtantiis, ut 400000 ad 1, hoc eſt, eſſe $\phi = \frac{1}{400000}$. Qui tamen valor vis ϕ ſi major vel minor poſteà deprehenſus fuerit, in eâdem ratione ſequentes omnes determinationes augendæ ſunt, vel minuendæ, adeoque ad juſtam menſuram facillimè reducentur. Erunt igitur

$K = -0.00000575$
$L = 0.00001643$
$M = 0.00000259$
$N = 0.00000090$
$O = 0.00000039$
$O' = 0.00000022$, &c.

Indeque colliguntur

$F = -0.00002459$
$G = 0.00002795$
$H = 0.00000345$
$I = 0.00000105$
$I' = 0.00000042$
&c.

atque reductis quantitatibus F, G, H, &c. in partes circuli, tandem habetur $s = w + 5''.07 \times \text{ſin.}\ \frac{1}{n} w - 5''.76 \times \text{ſin.}\ \frac{2}{n} w - 0''.71 \times \text{ſin.}\ \frac{3}{n} w - 0''.22 \times \text{ſin}\ \frac{4}{n} w -$, &c. ubi s denotat motum Terræ verum, w motum medium, et $\frac{1}{n} w$ angulum PSQ, ſive differentiam longitudinum heliocentricarum Terræ et Veneris.

Inde computatur ſequens tabula exhibens æquationem motûs Solis pro variâ diſtantiâ Veneris à Terrâ quam metitur angulus PSQ, ſive pro variâ differentiâ longitudinum heliocentricarum Terræ et Veneris quam metitur arcus circuli maximi inter Terram et Venerem interjectus et ſecundum ſeriem ſignorum à loco Terræ computatus.

Diff. long. hel. Terræ et Ven.	Æquatio motûs Solis.	Diff. long. hel. Terræ et Ven.	Æquatio motûs Solis.
°	″	°	″
Sig. 0. 0	— 0	Sig. VI. 0	— 0
10	1.6	10	2.6
20	2.8	20	5.0
30	3.4	30	7.0
Sig. I. 10	3.1	Sig. VII. 10	8.4
20	2.1	20	9.1
30	0.4	30	9.2
Sig. II. 10	+ 1.6	Sig. VIII. 10	8.6
20	3.8	20	7.5
30	5.8	30	5.8
Sig. III. 10	7.5	Sig. IX. 10	3.8
20	8 6	20	1.6
30	9.2	30	+ 0.4
Sig. IV. 10	9.1	Sig. X. 10	2.1
20	8.4	20	3.1
30	7.0	30	3.4
Sig. V. 10	5.0	Sig. XI. 10	2.8
20	2.6	20	0.6
30	0.	30	0

COROLL. II.

Si tellus gravitate ſuâ in Solem in circulo revolvi poſſe ſupponatur, adveniente Veneris actione variari debere diſtantiam ejus à Sole patet ex hac propoſitione. Eſto angulus $\frac{1}{n}s$, ſeu PSQ $= 90°$, vel $270°$, atque æquatio generalis $x = 1 - Q + K \cos. \frac{1}{n}s + L \cos. \frac{2}{n}s + M \cos. \frac{3}{n}s +$, &c. in hanc abit $x = 0.9999693$; et ſi ſit PSQ $= 180°$, fit $x = 1.0000053$.

Unde ſi diſtantia Terræ à Sole, ubi verſatur in conjunctione cum Venere, ponatur - - - - - - - - - -	10000000
In quadraturis cum Venere erit ipſius diſtantia - - - - - - - - - - -	9999693
Atque in oppoſitione - - - - - - -	10000053

PROPOSITIO VII. PROBLEMA.

In ſyſtemate duorum planetarum in circulis circa Solem revolventium, motum nodorum orbis planetæ exterioris in plano orbis planetæ interioris inveſtigare.

Eſto P locus planetæ exterioris (Fig. 5.) in orbe ſuo PN, SQ recta conjungens Solem et planetam interiorem, et dicatur c ſinus inclinationis duorum orbium ad ſe invicem ad radium 1, atque per propoſitionem quintam eſt $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$ vis qua planeta P amovetur ab orbe ſuo ſecundum directionem parallelam rectæ SQ, hujuſque vis ea pars quæ perpendiculariter agit in planum orbis PN, per ſimile ratiocinium quo uſi ſumus in Prop. III. prodit æqualis $c \times$ ſin. QN $\times \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}}$, et motus interſectionis plani orbis PN cum plano orbis QN fit $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. PN $\times$ ſin. QN $\times$ Pp quo tempore planeta P deſcribit in orbe ſuo arcum quàm minimum Pp.

Deinde ſi deſignaverit D locum planetæ P ubi verſatur in conjunctione cum planetâ interiore, et ponantur DP $= s$, P$p = \dot{s}$, DN $= a$, erit PN $= s + a$, QN $= s + \frac{1}{n} s + a$ quamproximè, atque ſin. PN $\times$ ſin. QN $= \frac{1}{2}$ coſ. $\frac{1}{n} s - \frac{1}{2}$ coſ. $\overline{2s + \frac{1}{n} s + 2a}$.

Unde, calculum proſequendo uti in propoſitione tertiâ, motus nodorum factus, quo tempore planeta P à loco conjunctionis D diſcedens deſcripſerit in orbe ſuo arcum quemlibet DP, exprimetur per

$$\frac{\varphi k n}{2t^3} \text{ in } \frac{S}{2n} s + \overline{R - \frac{t^3}{k^3} + \frac{T}{2}} \times \text{ſin.}\ \frac{1}{n} s + \frac{S+V}{4} \text{ ſin.}\ \frac{2}{n} s$$

$$+ \frac{T+W}{6} \text{ ſin.}\ \frac{3}{n} s + \frac{V+X}{8} \text{ ſin.}\ \frac{4}{n} s +, \text{ \&c.}$$

$$+ \frac{\varphi k n}{2t^3} \text{ in } Z \times \text{ſin.}\ 2a - \overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n+1} \text{ſin.}\ \overline{2s + \frac{1}{n} s + 2a}$$

$$- \frac{S}{2} \times \frac{1}{2n} \text{ ſin.}\ \overline{2s + 2a} - \frac{S}{2} \times \frac{1}{2n+2} \text{ ſin.}\ \overline{2s + \frac{2}{n} s + 2a}$$

$$- \frac{T}{2} \times \frac{1}{2n-1} \text{ ſin.}\ \overline{2s - \frac{1}{n} s + 2a} - \frac{T}{2} \times \frac{1}{2n+3} \text{ ſin.}$$

$$\overline{2s + \frac{3}{n} s + 2a} - \frac{V}{2} \times \frac{1}{2n-2} \text{ ſin.}\ \overline{2s - \frac{2}{n} s + 2a}$$

$- \frac{V}{2} \times \frac{1}{2n+4}\ \text{ſin.}\ \overline{2s + \frac{4}{n}s + 2a} - \frac{W}{2} \times \frac{1}{2n-3}\ \text{ſin.}\ \overline{2s - \frac{3}{n}s + 2a} - \frac{W}{2} \times \frac{1}{2n+5}\ \text{ſin.}\ \overline{2s + \frac{5}{n}s + 2a}$, &c. exiſtente $Z = 2n + 1$ in $\overline{R - \frac{t^3}{k}} \times \frac{1}{\overline{2n+1}|^2} + \frac{S}{2n \times \overline{2n+2}} + \frac{T}{\overline{2n-1} \times \overline{2n+3}} + \frac{V}{\overline{2n-2} \times \overline{2n+4}} + \frac{W}{\overline{2n-3} \times \overline{2n+5}} +$, &c. In quibus ſeriebus manifeſta eſt terminorum progreſſio. Q. E. I.

COROLL.

Hinc in conjunctionibus expreſſio motûs nodi evadit $\frac{\varphi k}{2t^3} \times \overline{\frac{S}{2}s - nZ \times \overline{\text{ſin.}\ \overline{2s + 2a} - \text{ſin.}\ 2a}}$. Hicque eſt motus nodi factus quo tempore planetæ P et Q à conjunctione procedentes ad conjunctionem quamvis aliam pervenerint, exhibente s arcum à planetâ P in ſuâ orbitâ intereà deſcriptum. Terminus $\frac{\varphi k}{2t^3} \times \frac{S}{2}s$ exprimit motum nodi medium, et terminus alter $\frac{\varphi kn}{2t^3} Z \times \overline{\text{ſin.}\ \overline{2s + 2a} - \text{ſin.}\ 2a}$ indicat æquationem *periodicam generalem*; vel etiam, ſi conjunctio illa à qua deſumitur computationis initium, fieri ſupponatur in nodo, vel propè ad nodum, æquatio periodica generalis fit $\frac{\varphi kn}{2t^3} Z \times \text{ſin}\ 2s$.

Deſignet jam planeta P Terram, Q Venerem, eritque poſt unam revolutionem ſynodicam, id eſt, poſt revolutionem Veneris ad Terram, $\frac{1}{n}s = 360°$, proindeque $s = n \times 360^\circ = 575^\circ\ 31'$. Quare motus nodi medius huic temporis ſpatio congruens fit $\frac{\phi k n}{4t^3} S \times 360^\circ$, qui imminutus in ratione revolutionis Terræ circa Solem ad ejuſdem revolutionem ad Venerem, hoc eſt, in ratione 1 ad n, evadit $\frac{\phi k}{4t^3} S \times 360^\circ = 5''.20$, motus ſcilicet nodi medius annuus quo regreditur interſectio planorum orbium Terræ ac Veneris; atque hic motus ſpatio centum annorum fit $8'\ 40''$.

In computo æquationis *periodicæ generalis* $\frac{\phi k n}{2t^3} Z \times$ ſin. $2s$, advertendum eſt omnes terminos, ex quibus componitur valor quantitatis Z, eoſdem hîc eſſe ac in Prop. III. præter terminum primum $\overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n+1}$ qui ob diverſum valorem quantitatum t et k diverſus eſt. Hîc igitur provenit $Z = 31.59$, adeoque $\frac{\phi k n}{2t^3} Z \times$ ſin. $2s = 5'' \times$ ſin. $2s$; unde patet æquationem hanc nunquam ſuperare $5''$. Motus igitur nodi verus, nimirùm $\frac{\phi k}{2t^3} \times \overline{\frac{S}{2} s - nZ \times \text{ſin. } 2s}$, peractâ unâ revolutione ſynodicâ poſt conjunctionem factam in nodo, evadit $8'.3 - 5'' \times$ ſin. $71^\circ.\ 2'$, quia tunc eſt ſin. $2s =$ ſin. $2 \times 575^\circ.\ 31' =$ ſin. $71^\circ.\ 2'$; et per ratiocinium ſimile ei, quod in Coroll. II. Prop. III. uſurpatum eſt, conſtabit $8''.3 - 5''.8 \times$ coſ. $\overline{2r - 1} \times 35^\circ.\ 31'$ exprimere regreſſum nodi factum tempore illius revolutionis ſynodicæ, cujus locum

cum in ſerie revolutionum indicat numerus r. Hinc computatur tabula ſequens quæ exhibet regreſſum nodi orbitæ Terreſtris in plano orbis Veneris pro duodecim ſigillatìm revolutionibus ſynodicis quæ proximè ſequuntur conjunctionem Terræ et Veneris factam in nodo, vel proximè ad nodum.

In revol. ſynod.	Regreſſus nodi Ter.	In revol. ſynod.	Regreſſus nodi Ter.
	$''$		$''$
1	4	7	9
2	10	8	14
3	14	9	11
4	10	10	4
5	4	11	3
6	3	12	9

Patet autem æquationem *periodicam ſpecialem*, nempe $5''.8 \times \text{coſ.}\ \overline{2r - 1} \times 35^\circ.\ 31'$, ubi maxima eſt, evadere $5''.8$, et regreſſum nodi in quavis revolutione Terræ ad Venerem non aſſurgere ultra $14''$, nec minui citra $2''\frac{1}{2}$.

PROPOSITIO VIII. PROBLEMA.

Iiſdem poſitis, variationem inclinationis orbis planetæ exterioris ad planum orbis planetæ interioris determinare.

Deſignet I variationem inclinationis factam quo tempore planeta P deſcribit arcum quàm minimum

Pp, et N motum nodi eodem tempore confectum, ac per ratiocinium omnino ſimile ei quod adhibitum eſt in propoſitione quartâ habetur $I = N \times \frac{c \times \text{coſ. } PN}{\text{ſin. } PN}$: ſed per propoſitionem præcedentem eſt $N = \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times \text{ſin. } PN \times \text{ſin. } QN \times Pp$, adeoque fit $I = \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times c \times \text{coſ. } PN \times \text{ſin. } QN \times Pp$.

Unde, cùm hîc ſit $PN = s + a$, $QN = s + \frac{1}{n}s + a$, proindeque coſ. $PN \times$ ſin. $QN = \frac{1}{2}\text{ ſin. } \frac{1}{n}s + \frac{1}{2}\text{ ſin. } \overline{2s + \frac{1}{n}s + 2a}$, ſumptâ fluente prodit variatio inclinationis genita, quo tempore planeta deſcripſerit in orbe ſuo arcum quemlibet DP à loco conjunctionis D, æqualis $\frac{\varphi c k n}{2t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{ſin. verſ. } \frac{1}{n}s + \frac{S - V}{4}\text{ ſin. verſ. } \frac{2}{n}s + \frac{T - W}{6}\text{ ſin. verſ. } \frac{3}{n}s + \frac{V - X}{8}\text{ ſin. verſ. } \frac{4}{n}s +$, &c. $+ \frac{\varphi c k n}{2t^3}$ in $-Z \times \text{ſin. verſ. } 2a + \overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n+1}\text{ ſin. verſ. } \overline{2s + \frac{1}{n}s + 2a} + \frac{S}{2} \times \frac{1}{2n}\text{ ſin. verſ. } \overline{2s + 2a} + \frac{S}{2}$ [$\times \frac{1}{2n+2}\text{ ſin. verſ. } \overline{2s + \frac{2}{n}s + 2a} + \frac{T}{2} \times \frac{1}{2n-1}\text{ ſin. verſ. } \overline{2s - \frac{1}{n}s + 2a} + \frac{T}{2} \times \frac{1}{2n+3}\text{ ſin. verſ. } \overline{2s + \frac{3}{n}s + 2a} + \frac{V}{2} \times \frac{1}{2n-2}\text{ ſin. verſ. } \overline{2s - \frac{2}{n}s + 2a}$

$+$

$+\frac{V}{2}\times\frac{1}{2n+4}$ ſin. verſ. $\overline{2s+\frac{4}{n}s+2a}$, &c. Eumdem hîc habet valorem quantitas Z ac in propoſitione præcedente. Q. E. I.

COROLL.

Ubi angulus PSQ eſt nullus, vel multiplex anguli 360°, id eſt, ubi planetæ verſantur in conjunctione, variatio inclinationis genita generatìm eſt $\frac{\varphi ckn}{2t^3}Z\times\overline{\text{ſin. verſ. }\overline{2s+2a}-\text{ſin. verſ. }2a}$ quæ, ſi ponatur arcus DN $= a = 0$, fit $\frac{\varphi ckn}{2t^3}Z\times$ ſin. verſ. $2s$.

Atque hoc eſt decrementum inclinationis orbis planetæ P ad orbem planetæ Q factum in qualibet ſerie revolutionum ad conjunctionem, initio ſumpto à conjunctione factâ in nodo, vel prope ad nodum, et deſignante s arcum intereà à planetâ P in orbe ſuo deſcriptum.

Si inde computetur decrementum inclinationis orbis Terreſtris ſupra planum orbitæ Veneris factum poſt quotcumque revolutiones Veneris ad Terram, fiet $\frac{\varphi ckn}{2t^3}Z\times$ ſin. verſ. $2s = 0''.3\times$ ſin. verſ. $2s$, adeoque hoc decrementum, ubi maximum evadit, non ſuperat $0''.6$, ac proinde in omni caſu negligi poteſt.